Online class # 01

Date: 22/06/2021

Chapter 04 (gradient)

Time: 0930 – 1040

Video: https://youtu.be/S-qluo4oC-0

$$\frac{dy}{dx} = ?$$

$\vec{A}(x) = \vec{A} = 3x^2\,\hat{i} + x^3\,\hat{j}$

$$\frac{d\vec{A}}{dx} = 6x\,\hat{i} + 3x^2\,\hat{j}$$

$\vec{B}(x,y)$

$\vec{B} = 3x^2\,\hat{i} + y^3\,\hat{j}$

x

$$\frac{d}{dx}$$

$$\frac{\partial}{\partial x}$$

Nabla

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$\rightarrow \phi(x,y,z)$, $\rightarrow \vec{A}(x,y,z)$

$\vec{A}\cdot\vec{B}$ $\quad \vec{A}\times\vec{B}$

$\phi(x, y, z)$

$$\nabla \phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

Gradient

grad ϕ, grad of ϕ, $\nabla \phi$

$$\phi(x, y, z) = 3x^2 y z^3$$

$$\nabla \phi = 6xyz^3\,\hat{i} + 3x^2 z^3\,\hat{j} + 9x^2 y z^2\,\hat{k}$$

Divergence $\vec{\nabla} \cdot \vec{A}$

$$\vec{A}(x,y,z) = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{A}(x,y,z) = \underline{x} \hat{i} + y \hat{j} + z \hat{k}$$

$$\underline{\vec{\nabla} \cdot \vec{A} = 3}$$

$$\vec{A}(x,y,z) = x^2 \hat{i} + y \hat{j} + z^3 \hat{k}$$

$$\vec{\nabla} \cdot \vec{A} = 2x + 1 + 3z^2$$

$$\vec{A} = x^2y\,\hat{i} + y^2z\,\hat{j} + z^2x\,\hat{k}$$

$$\vec{\nabla}\cdot\vec{A} = 2(xy + yz + zx)$$

$$\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$$

$$\vec{\nabla}\cdot\vec{A} \neq \cancel{\vec{A}\cdot\vec{\nabla}} = A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}$$

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{A}(x,y,z) = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Curl $\vec{\nabla}\times\vec{A}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$\vec{\nabla}\cdot\vec{A}$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\vec{\nabla}(\phi + \psi) = \vec{\nabla}\phi + \vec{\nabla}\psi$$

$$\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B}$$

$$\vec{\nabla}\times(\vec{A}+\vec{B}) = \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B}$$

$$\boxed{\vec{\nabla}\cdot(\phi\vec{A}) = (\vec{\nabla}\phi)\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})}$$

$$\phi(x,y,z) = x^2 y z^3$$

$$\vec{A} = x^2 y\, i + xy^2 z\, j + z x k$$

1. $\phi(x,y,z) = 3x^2y - y^3z^2$

$\vec{\nabla}\phi \rightarrow (1, -2, -1)$

$\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$

$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\left(3x^2y - y^3z^2\right) = 6xy$

$\frac{\partial\phi}{\partial y} = 3x^2 - 3y^2z^2$

$\frac{\partial\phi}{\partial z} = 0 - 2y^3z$

$(1, -2, -1)$

$\frac{\partial\phi}{\partial x} = -12$

$\frac{\partial\phi}{\partial y} = 3 - 3(4)\cdot 1 = -9$

$\frac{\partial\phi}{\partial z} = -16$

$-12\hat{i} - 9\hat{j} - 16\hat{k}$

1-6, 10, 12

42 - 52

H.W.

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3

3: $\vec{r} = xi + yj + zk$

$$|\vec{r}| = r = (x^2+y^2+z^2)^{1/2} = \sqrt{x^2+y^2+z^2}$$

$$\phi = \ln|\vec{r}| = \ln(x^2+y^2+z^2)^{1/2}$$

$$= \frac{1}{2}\ln(x^2+y^2+z^2)$$

$$\nabla\phi = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z}$$

$$\frac{\partial\phi}{\partial x} = \frac{1}{2}\,\frac{1}{x^2+y^2+z^2}(2x+0+0)$$

$$= \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial f(\underline{x})}{\partial x} = g(r) \cdot \frac{\partial r}{\partial x}$$