Online class # 02

Date: 29/06/2021

Chapter 04 (gradient, divergence)

Time: 0930 – 1010

Video: https://youtu.be/qtSOQ_E8RII

$$\left(\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k\right) \cdot (dx\, i + dy\, j + dz\, k) = 0$$

$$\phi(x, y, z) = C$$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz = 0$$

$$z = f(x, y)$$

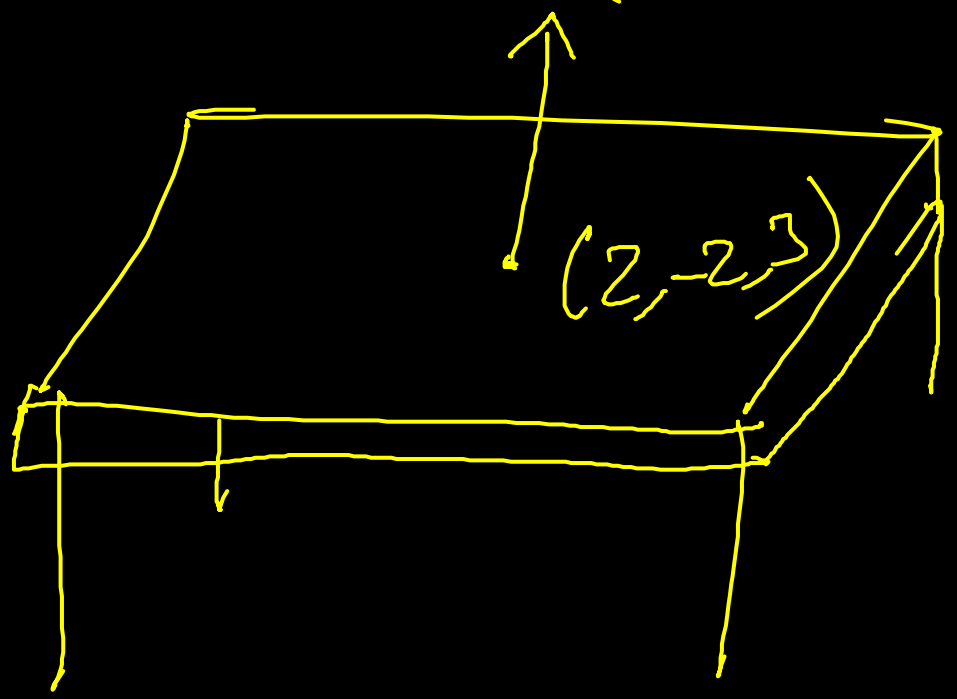
$$\frac{dz}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx}$$

$$\underline{dz} = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dz$$

$$\frac{dz}{dy} = \frac{\partial f}{\partial y}$$

$$dz = \frac{\partial f}{\partial y}dy$$

6.



$\vec{F}$

$\hat{n}$

$x^2y + 2xz = 1$

$|\vec{n}| = 1$

48 If $\boxed{\nabla V = 2r^4 \vec{r}}$ find V.

$$V = \int_a^b 2r^4 \vec{r}\, dr$$

$$= 2\int_a^b (x^2+y^2+z^2)^2 (x\hat{i}+y\hat{j}+z\hat{k})\, dx\, dy\, dz$$

$$= 2\int_a^b \int_b^c \int_c^a (x^2+y^2+z^2)^2 \, x\, dx$$

$$= 2\int_a^b \int_b^c \int_c^a (x^4+y^4+z^4+2x^2y^2+2y^2z^2+2z^2x^2)\, x\, dx$$

4 $\vec{\nabla} r^n = n r^{n-2} \vec{r}$

$\vec{\nabla} U = 2r^4 \vec{r}$

$\vec{\nabla} U = \frac{1}{3} 6 r^{6-2} \vec{r}$

$= \frac{1}{3} \vec{\nabla} r^6$

$\therefore U = \frac{1}{3} r^6 + C$

$n-2 = 4$

$n = 6$

$U(r) = \frac{6 r^6}{3} + C$

$U(0) = 5 = C$

Divergence :

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$\nabla\phi$ ← $\vec{\nabla}\cdot\vec{A}$ ← $\vec{\nabla}\times\vec{A}$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$A_x(x, y, z)$

15 $\vec{A} = x^2 z\,\hat{i} - 2y^3 z^2\,\hat{j} + xy^2 z\,\hat{k}$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial}{\partial x}(x^2 z) - \frac{\partial}{\partial y}(2y^3 z^2) + \frac{\partial}{\partial z}(xy^2 z)$$

15-19

$$= 2xz - 6y^2 z^2 + xy^2$$

$$\vec{\nabla}\cdot\vec{A}\Big|_{(1,-1,1)} = 2\cdot 1\cdot 1 - 6(-1)^2\cdot 1 + 1\cdot(-1)^2$$

$$= ??$$