Online class # 03

Date: 04/07/2021

Chapter 04 (divergence, curl)

Time: 0930 – 1025

Video: https://youtu.be/tm_4CfiwOrw

19. $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) = 0$

$\vec{\nabla} \cdot (r^{-3}\vec{r}) = (\vec{\nabla} r^{-3}) \cdot \vec{r} + r^{-3}\, \vec{\nabla} \cdot \vec{r}$

$= -3r^{-5}\, \vec{r} \cdot \vec{r} + r^{-3} \cdot 3$

$= -3r^{-5}\, r^2 + 3r^{-3}$

$= -3r^{-3} + 3r^{-3} = 0$

$\vec{\nabla} \cdot (\phi \vec{A})$

$= (\vec{\nabla}\phi) \cdot \vec{A}$

$+ \phi(\vec{\nabla} \cdot \vec{A})$

$\vec{\nabla}(r^n) = n r^{n-2}\, \vec{r}$

73 $\nabla^2(\ln r) = ?$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = |\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$

$\ln r = \ln(x^2 + y^2 + z^2)^{1/2}$

$= \frac{1}{2}\ln(x^2 + y^2 + z^2)$

$\nabla^2(\ln r) = \vec{\nabla}\cdot\vec{\nabla}(\ln r)$

$\vec{\nabla}(\ln r) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\ln r$

$\nabla^2(\ln r) = \vec{\nabla}\cdot\vec{\nabla}(\ln r)$

$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\ln r = \frac{\partial^2}{\partial x^2}\ln r + \frac{\partial^2}{\partial y^2}\ln r + \frac{\partial^2}{\partial z^2}\ln r$

$$\ln r = \frac{1}{2}\ln(x^2+y^2+z^2)$$

$$\frac{\partial^2}{\partial x^2}\ln r$$

$$\frac{\partial}{\partial x}\ln r = \frac{1}{2}\cdot\frac{2x}{x^2+y^2+z^2} = \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial^2}{\partial x^2}\ln r = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\ln r\right) = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2+z^2}\right)$$

$$= \frac{(x^2+y^2+z^2)\,1 - x\,(2x+0+0)}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2}{\partial x^2}\ln r = \frac{-x^2+y^2+z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2}{\partial y^2}\ln r = \frac{x^2-y^2+z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2}{\partial z^2}\ln r = \frac{x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$$

$$\nabla^2 \ln r = \frac{\partial^2}{\partial x^2}\ln r + \frac{\partial^2}{\partial y^2}\ln r + \frac{\partial^2}{\partial z^2}\ln r$$

$$= \frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}$$

74 $\nabla^2 r^n = n(n+1) r^{n-2}$, $n \to$ constant

We know $\boxed{\vec{\nabla} r^n = n r^{n-2}\vec{r}}$

$$\vec{\nabla}\cdot(\vec{\nabla} r^n) = \vec{\nabla}\cdot\left(n r^{n-2}\vec{r}\right) = n\,\vec{\nabla}\cdot\left(r^{n-2}\vec{r}\right)$$

$$= n\,\vec{\nabla}(r^{n-2})\cdot\vec{r} + n r^{n-2}\,\vec{\nabla}\cdot\vec{r}$$

$$= n(n-2) r^{n-4}\,\vec{r}\cdot\vec{r} + n r^{n-2}\cdot 3$$

$$= n(n-2) r^{n-2} + 3n r^{n-2} = n(n-2+3) r^{n-2}$$

$$= n(n+1) r^{n-2}$$

Curl

$$\vec{A} = iA_1 + jA_2 + kA_3$$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= i\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - j\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + k\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

123 $\vec{A} = xz^3 i - 2x^2yz j + 2yz^4 k$

$(1,-1,1)$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix}$$

$$= i(2z^4 + 2x^2y) - j(0 - 3xz^2) + k(-4xyz - 0)$$

$$= i(2z^4 + 2x^2y) + j\,3xz^2 - k\,4xyz$$

$$\nabla \times \bar{A}\Big|_{(1,-1,1)} = 0 + 3\hat{j} + 4\hat{k}$$

$$\nabla \times (\bar{A} + \bar{B}) = \nabla \times \vec{A} + \nabla \times \bar{B}$$

$$\underline{\nabla \times (\phi \vec{A})} = (\nabla \phi) \times \vec{A} + \phi (\nabla \times \vec{A})$$

26 $\vec{\nabla}\cdot(\vec{A}\times\vec{r}) = ?$

$\vec{\nabla}\times\vec{A} = 0$

$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{A}\times\vec{r} = \hat{i}(zA_2 - yA_3) - \hat{j}(zA_1 - xA_3) + \hat{k}(yA_1 - xA_2)$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{r}) = \frac{\partial}{\partial x}(zA_2 - yA_3) - \frac{\partial}{\partial y}(zA_1 - xA_3) + \frac{\partial}{\partial z}(yA_1 - xA_2)$$

$$\nabla\cdot(\vec{A}\times\vec{r}) = z\frac{\partial A_2}{\partial x} - y\frac{\partial A_3}{\partial x} - z\frac{\partial A_1}{\partial y} + x\frac{\partial A_3}{\partial y} + y\frac{\partial A_1}{\partial z} - x\frac{\partial A_2}{\partial z}$$

$$= x\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - y\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + z\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$$= (x\hat{i} + y\hat{j} + z\hat{k})\left\{\hat{i}(\;) - \hat{j}(\;) + \hat{k}(\;)\right\}$$

$$= \bar{r}\cdot(\vec{\nabla} \times \vec{A}) = 0$$

$$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

$$= A_1(x, y, z)$$

$$A_1 = xyz$$

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{\nabla}\cdot\bar{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

$$\frac{\partial A_1}{\partial y} = xz$$

$$\frac{\partial (yA_1)}{\partial x} = y\frac{\partial A_1}{\partial x}$$