Online class # 04

Date: 06/07/2021

Chapter 04 (Gradient, Divergence, Curl)

Time: 0935 – 1040

Video: https://youtu.be/YKzAmwC8MkQ

51. If $\nabla\phi = 2xyz^3\,\mathbf{i} + x^2z^3\,\mathbf{j} + 3x^2yz^2\,\mathbf{k}$,

find $\phi(x,y,z)$ if $\phi(1,-2,2) = 4$.

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$\frac{\partial\phi}{\partial x} = 2xyz^3$$

$$\phi = \boxed{x^2yz^3} + f(z,y)$$

$$\frac{\partial\phi}{\partial y} = x^2z^3$$

$$\phi = x^2yz^3 + f(x,z)$$

$$\phi = x^2yz^3 + f(x,y)$$

51. If $\vec{\nabla}\varphi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$

find $\varphi(x,y,z)$ if $\varphi(1,-2,2) = 4$.

Solve:

$$\vec{\nabla}\varphi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$

$$\Rightarrow \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\varphi = \hat{i}\frac{\delta}{\delta x}(x^2yz^3) + \hat{j}\frac{\delta}{\delta y}(x^2yz^3) + \hat{k}\frac{\delta}{\delta z}(x^2yz^3)$$

$$\Rightarrow \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\varphi = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)(x^2yz^3)$$

$$\Rightarrow \varphi = x^2yz^3 + c$$

$$\therefore \varphi(x,y,z) = x^2yz^3 + c$$

$$\therefore \varphi(1,-2,2) = (1)^2(-2)(2)^3 + c = -16 + c$$

Now, $\varphi(1,-2,2) = 4$

$$\Rightarrow -16 + c = 4$$

$$\therefore c = 20$$

$$\therefore \varphi(x,y,z) = x^2yz^3 + 20.$$

52. If $\nabla\psi = (y^2 - 2xyz^3)\mathbf{i} + (3 + 2xy - x^2z^3)\mathbf{j} + (6z^3 - 3x^2yz^2)\mathbf{k}$, find ψ.

$$\nabla\psi = i\frac{\partial\psi}{\partial x} + j\frac{\partial\psi}{\partial y} + k\frac{\partial\psi}{\partial z}$$

$$\frac{\partial\psi}{\partial x} = y^2 - 2xyz^3 \rightarrow \psi = xy^2 - x^2yz^3 + f(y,z)$$

$$\frac{\partial\psi}{\partial y} = 3 + 2xy - x^2z^3 \rightarrow \psi = 3y + xy^2 - x^2yz^3 + f(x,z)$$

$$\frac{\partial\psi}{\partial z} = 6z^3 - 3x^2yz^2 \rightarrow \psi = \frac{3z^4}{2} - x^2yz^3 + f(x,y)$$

c

$\psi =$

$$\psi = xy^2 - x^2yz^3 + 3y + \frac{3z^4}{2} + c$$

K

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$A_x = B_x \quad \Big| \quad A_y = B_y$$

$$A_z = B_z$$

73. Evaluate $\nabla^2(\ln r)$.

Gradient, divergence, Curl.

73.

$$\nabla^2(\ln r)$$

$$= \vec{\nabla}\cdot\vec{\nabla}(\ln r)$$

$$= \vec{\nabla}\cdot\left(\frac{1}{r}\cdot\hat{r}\right)$$

$$= \vec{\nabla}\cdot\left(\frac{1}{r}\cdot\frac{\vec{r}}{r}\right)$$

$$= \vec{\nabla}\left(\frac{\vec{r}}{r^2}\right)$$

$$= \vec{\nabla}\left(\frac{1}{r^2}\cdot\vec{r}\right)$$

$$= \left(\vec{\nabla}\frac{1}{r^2}\right)\cdot\vec{r} + \frac{1}{r^2}\left(\vec{\nabla}\cdot\vec{r}\right)$$

$$= \left(\frac{-2}{r^3}\cdot\hat{r}\right)\cdot\vec{r} + \frac{1}{r^2}\left\{\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\right\}$$

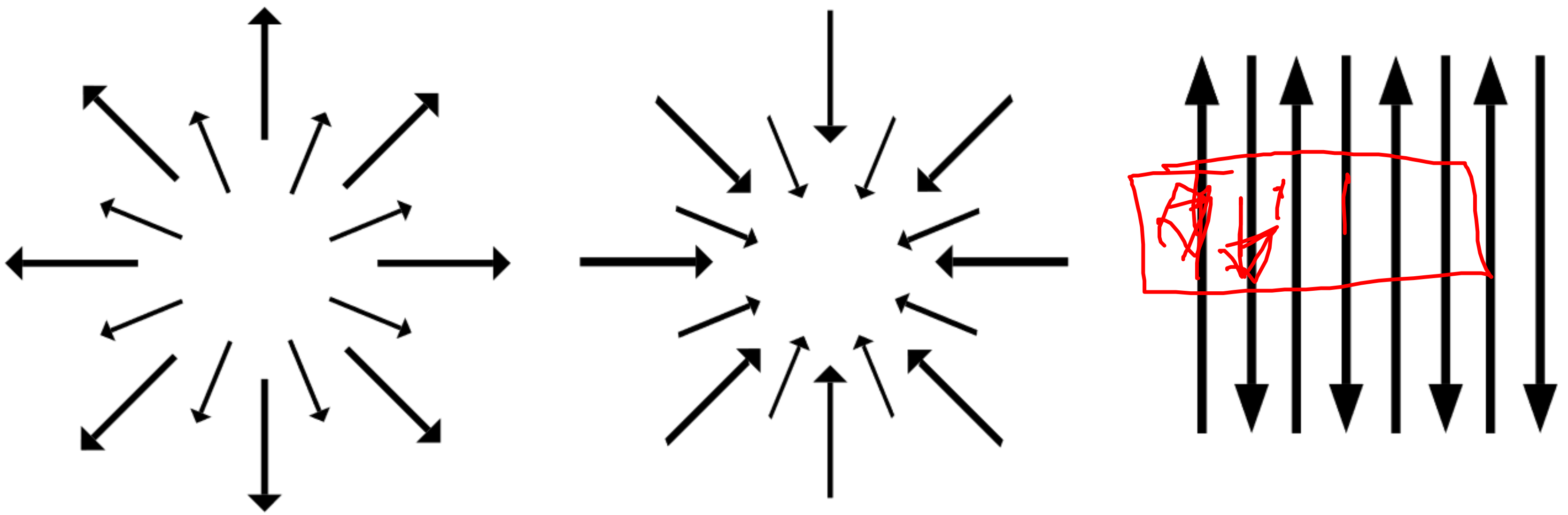
$$= -\frac{2}{r^4}\cdot r^2 + \frac{1}{r^2}(1+1+1)$$

$$= -\frac{2}{r^2} + \frac{3}{r^2}$$

$$= \frac{1}{r^2}$$

(Ans.)

Divergence of vector field



$\partial/\partial x(\mathbf{V}_x) > 0$
$\partial/\partial y(\mathbf{V}_y) > 0$
$\nabla\cdot(\mathbf{V}) > 0$

$\partial/\partial x(\mathbf{V}_x) < 0$
$\partial/\partial y(\mathbf{V}_y) < 0$
$\nabla\cdot(\mathbf{V}) < 0$

$\partial/\partial x(\mathbf{V}_x) = 0$
$\partial/\partial y(\mathbf{V}_y) = 0$
$\nabla\cdot(\mathbf{V}) = 0$

104. If $\mathbf{A}$ and $\mathbf{B}$ are irrotational, prove that $\mathbf{A} \times \mathbf{B}$ is solenoidal.

99. Prove $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$.

59. Find the unit outward drawn normal to the surface $(x-1)^2+y^2+(z+2)^2 = 9$ at the point $(3,1,-4)$.
Ans. $(2\mathbf{i}+\mathbf{j}-2\mathbf{k})/3$

60. Find an equation for the tangent plane to the surface $xz^2+x^2y = z-1$ at the point $(1,-3,2)$.
Ans. $2x-y-3z+1 = 0$

61. Find equations for the tangent plane and normal line to the surface $z = x^2+y^2$ at the point $(2,-1,5)$.
Ans. $4x-2y-z = 5$, $\dfrac{x-2}{4} = \dfrac{y+1}{-2} = \dfrac{z-5}{-1}$ or $x = 4t+2$, $y = -2t-1$, $z = -t+5$

62. Find the directional derivative of $\phi = 4xz^3-3x^2y^2z$ at $(2,-1,2)$ in the direction $2\mathbf{i}-3\mathbf{j}+6\mathbf{k}$.
Ans. $376/7$

63. Find the directional derivative of $P = 4e^{2x-y+z}$ at the point $(1,1,-1)$ in a direction toward the point $(-3,5,6)$. *Ans.* $-20/9$

$\cancel{\phi = (x\ \ )}$ $\phi = (x-1)^2$

$$\phi = xz^2 + x^2y - z$$

$(1,-3,2)$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$= i(z^2+2xy) + j(x^2) + k(2xz-1)$$

$$\nabla\phi(1,-3,2) = -2i + j + 3k$$

$$[(xi+yj+zk) - (i-3j+2k)]\cdot(-2i+j+3k) = 0$$

$ai + bj + ck$

(x_0, y_0, z_0)

$\left[(xi + yj + zk) - (x_0 i + y_0 j + z_0 k) \right] \cdot$

$(ai + bj + ck) = 0$

$$\{(x-1)i+(y+3)j+(z-2)k\}\cdot(-2i+j+3k)=0$$

$$\Rightarrow -2(x-1)+(y+3)+3(z-2)=0$$

$$\Rightarrow -2x+2+y+3+3z-6=0$$

$$\Rightarrow -2x+y+3z-1=0$$

$$2x-y-3z+1=0$$

65. $\phi = axy^2 + byz + cz^2x^3$ $\quad (1, 2, -1)$

$$\nabla\phi = \hat{i}(ay^2 + 3cz^2x^2) + \hat{j}(2axy + bz) + \hat{k}(by + 2czx^3)$$

$$\text{or } \nabla\phi\Big|_{(1,2,-1)} = \hat{i}(4a+3c) + \hat{j}(4a-b) + \hat{k}(2b-2c)$$

$$\text{or } \begin{cases} 4a+3c = 0 \\ 4a-b = 0 \end{cases} \quad \Big| \quad 2b - 2c = 64$$

$3c + b = 0$

$b = -3c$

$= -3(-8)$

$b = 24$

$4a - b = 6$

$4a = 24$

$a = 6$

$2b - 2c = 64$

$b - c = 32$

$-3c - c = 32$

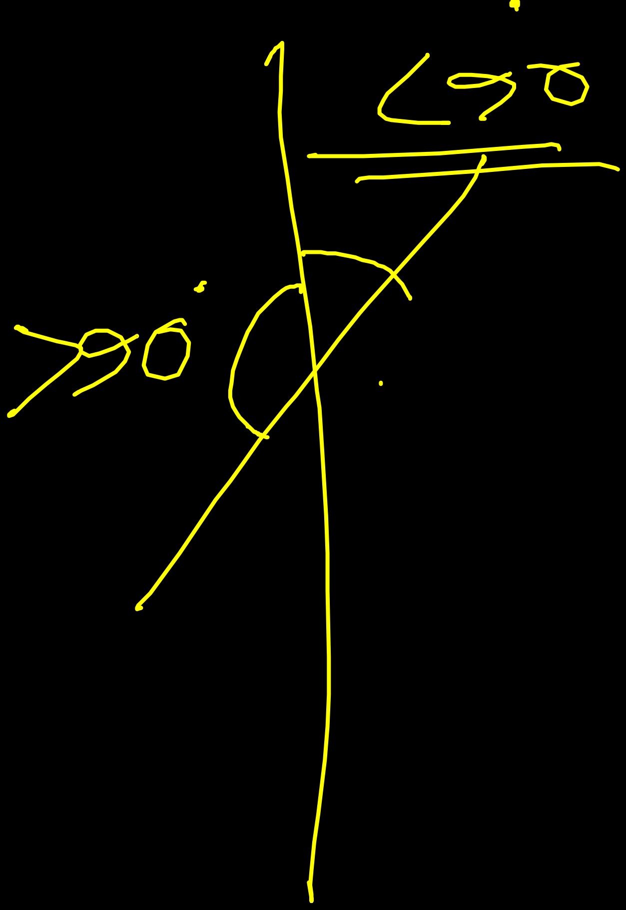
$-4c = 32$

$c = -8$

$$\theta = 95$$

$$180 - \theta$$

$$\boxed{\theta = 85^\circ}$$



$$\bar{A} \cdot \bar{B} = AB\cos\theta = 0$$

$$\theta = 90^\circ$$