Online class # 05

Date: 11/07/2021

Chapter 04 (Gradient, Divergence, Curl)

Time: 0930 – 1025

Video: https://youtu.be/tyHdhNe5DqY

Q6 $f(r)$, $\vec{\nabla} f(r)$.

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$\cancel{f(x,y,z)}$ $f(r)$, $r(x,y,z)$

$$\frac{\partial f(r)}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x}$$

$$= f'(r)\frac{\partial r}{\partial x}$$

$$\vec{r} = xi + yj + zk$$

$$|\vec{r}| = r = (x^2+y^2+z^2)^{1/2}$$

$$\vec{\nabla} f(r) = i f'(r)\frac{\partial r}{\partial x} + j f'(r)\frac{\partial r}{\partial y} + k f'(r)\frac{\partial r}{\partial z}$$

$$= i f'(r)\frac{x}{r} + j f'(r)\frac{y}{r} + k f'(r)\frac{z}{r}$$

$$r = (x^2+y^2+z^2)^{1/2}$$

$$\frac{\partial r}{\partial x} = \frac{1}{2}(x^2+y^2+z^2)^{\frac{1}{2}-1} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot 2x$$

$$= \frac{x}{(x^2+y^2+z^2)^{1/2}} = \frac{x}{r}$$

$$\frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\vec{\nabla} f(r) = f'(r) \cdot \frac{1}{r}\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$= f'(r)\,\frac{\vec{r}}{r}.$$

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$$f(r) = 3r^{2} - 4r^{1/2} + 6r^{-\frac{1}{3}}$$

$$\nabla(r^{n}) = n\,r^{n-2}\,\vec{r}$$

$$\nabla f(r) = 3\,\nabla(r^{2}) - 4\,\nabla\left(r^{1/2}\right) + 6\,\nabla\left(r^{-1/3}\right)$$

$$= 3\cdot 2\,r^{2-2}\cdot\vec{r} - 4\cdot\frac{1}{2}\,r^{\frac{1}{2}-2}\,\vec{r} + 6\left(-\frac{1}{3}\right)r^{-\frac{1}{3}-2}\,\vec{r}$$

$$= \left(6 - 2r^{-3/2} - 2\,r^{-7/3}\right)\vec{r}$$

(44) $F = x^2 z + e^{y/x}$

$G = 2z^2 y - xy^2$

$M = FG = 2x^2yz^2 - x^2y^2z + 2z^2 y e^{y/x} - xy^2 e^{y/x}$

$\nabla M = ?$

8^{0}

$$\vec{\nabla} \cdot \left[r \, \vec{\nabla}\left(\frac{1}{r^3}\right) \right] = ?$$

$$\vec{\nabla}(r^n) = n r^{n-2} \vec{r}$$

$$\nabla\left(\frac{1}{r^3}\right) = \nabla\left(r^{-3}\right)$$

$$= -3 r^{-5} \vec{r}$$

$$= \vec{\nabla} \cdot \left[r \left(-3 r^{-5} \vec{r} \right) \right]$$

$$= -3 \, \vec{\nabla} \cdot \left(r^{-4} \vec{r} \right)$$

$$\vec{\nabla} \cdot (\phi A) = (\vec{\nabla}\phi) \cdot A + \phi \, \vec{\nabla} \cdot \vec{A}$$

9) $\vec\nabla \times \left(\dfrac{\vec r}{r^2} \right) = ?$

$$\vec\nabla \times (\phi \vec A) = (\vec\nabla \phi) \times \vec A + \phi\, \vec\nabla \times \vec A$$

$$= \vec\nabla \times \left(r^{-2} \vec r \right)$$

$$= \left(\vec\nabla r^{-2}\right) \times \vec r + r^{-2} \left(\vec\nabla \times \vec r \right)$$

$$= -2 r^{-4}\, \underline{\vec r \times \vec r} + r^{-2} \cdot 0$$

$$= 0$$

$\vec\nabla \times \vec r =$

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$$\bar{A} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$$

$$\bar{A} = \vec{\nabla}\phi = i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y} + k\frac{\partial \phi}{\partial z}$$

$$\frac{\partial \phi}{\partial x} = 6xy + z^3$$

$$\phi = 3x^2y + z^3 x + f(y, z)$$

$$\frac{\partial\phi}{\partial y} = 3x^2 - z$$

$$\phi = 3x^2 y - yz + f(z,x)$$

$$\frac{\partial\phi}{\partial z} = 3xz^2 - y$$

$$\phi = \cancel{3}\, xz^3 - yz + f(x,y)$$

$$\phi = 3x^2 y + z^3 x - yz + \text{const.}$$

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$$E = \frac{\vec{r}}{r^2} = i\,\frac{x}{(x^2+y^2+z^2)} + j\,\frac{y}{(x^2+y^2+z^2)} + k\,\frac{z}{(x^2+y^2+z^2)}$$

$$E = -\nabla\phi = -i\,\frac{\partial\phi}{\partial x} - j\,\frac{\partial\phi}{\partial y} - k\,\frac{\partial\phi}{\partial z}$$

$$\boxed{\frac{\partial\phi}{\partial x}} = -\frac{x}{(x^2+y^2+z^2)}$$

$$\phi = -\int \frac{x}{(x^2+y^2+z^2)}\,dx$$

$$x^2+y^2+z^2 = p$$

$$d(x^2+y^2+z^2) = dp$$

$$2x\,dx = dp$$

$$x\,dx = \frac{1}{2}dp$$

$$= -\frac{1}{2}\int \frac{dp}{p}$$

$$= -\frac{1}{2}\ln p + C$$

$$= -\frac{1}{2}\ln(x^2+y^2+z^2) + C$$

$$\phi = \ln\left(x^2+y^2+z^2\right)^{-1/2} + C$$

$$= \ln\left(\frac{1}{r}\right) + f(y,z)$$

$$\frac{\partial\phi}{\partial z} \longrightarrow \phi$$

$$\frac{\partial\phi}{\partial y} \longrightarrow \phi$$

$$\phi(a) = 0$$

$$\phi = \ln\left(\frac{1}{r}\right) + C$$

$$\phi(a) = \ln(1/a) + C = 0 \Rightarrow C = -\ln\left(\frac{1}{a}\right)$$

$$= \ln a$$

$$\phi = \ln\left(\frac{1}{r}\right) + \ln a$$

$$= \ln a + \ln \frac{1}{r}$$

$$= \ln\left(\frac{a}{r}\right) \quad \boxed{}$$

$$\vec{V} = iV_1 + jV_2 + kV_3$$

$$\nabla \times \vec{V} = i\left(\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z}\right) + j\left(\frac{\partial V_1}{\partial z} - \frac{\partial V_3}{\partial x}\right) + k\left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y}\right)$$

$$\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z} = 2$$

$$\frac{\partial V_1}{\partial z} - \frac{\partial V_3}{\partial x} = 1$$

$$\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} = 3$$

$$\nabla \times \vec{V} = 2i + j + 3k$$