Online class # 13

Date: 24/08/2021

Chapter 06 (Divergence theorem, Stokes' theorem)

Time: 0915 – 1025

Video: https://youtu.be/MImEcVnDm_o

Divergence theorem, Stokes' theorem, and Related integral theorem

Chapter 6

$$\iint_S \nabla \times \bar{A} \cdot \hat{n}\, ds = \oint_C \bar{A} \cdot d\bar{r}$$

$$\bar{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}$$

$$\nabla \times \bar{A} = (\nabla \times A_1 \hat{i}) + (\nabla \times A_2 \hat{j}) + (\nabla \times A_3 \hat{k})$$

$$\nabla \times (A_1 \hat{i}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & 0 & 0 \end{vmatrix}$$

$$= \frac{\partial A_1}{\partial z} \hat{j} - \frac{\partial A_1}{\partial y} \hat{k}$$

$$\iint_S \{\vec\nabla\times(A_1\hat i) + \vec\nabla\times(A_2\hat j) + \vec\nabla\times(A_3\hat k)\}\cdot\hat n\, ds$$

$$= \oint_C (A_1\hat i + A_2\hat j + A_3\hat k)\cdot(dx\,\hat i + dy\,\hat j + dz\,\hat k)$$

$$= \oint_C (A_1 dx + A_2 dy + A_3 dz)$$

$$\iint_S \nabla \times (A_1 i) \cdot \hat{n}\, ds = \iint_S \left(\frac{\partial A_1}{\partial z} j - \frac{\partial A_1}{\partial y} k \right) \cdot \hat{n}\, ds$$

$$\boxed{2x + y + 3z = 5}$$

$$y = 5 - 2x - 3z = f_y(x, z)$$

$$x = \frac{1}{2}(5 - y - 3z) = f_x(y, z)$$

$$z = \frac{1}{3}(5 - 2x - y) = f_z(x, y)$$

$$z = f(x,y)$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= x\hat{i} + y\hat{j} + f(x,y)\hat{k}$$

$$\frac{\partial \vec{r}}{\partial y} = 0 + \hat{j} + \frac{\partial f}{\partial y}\hat{k}$$

$$\frac{\partial \vec{r}}{\partial y}\cdot\hat{n} = 0$$

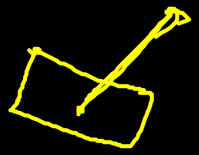
$$\hat{j}\cdot\hat{n} = -\frac{\partial f}{\partial y}\hat{k}\cdot\hat{n}$$

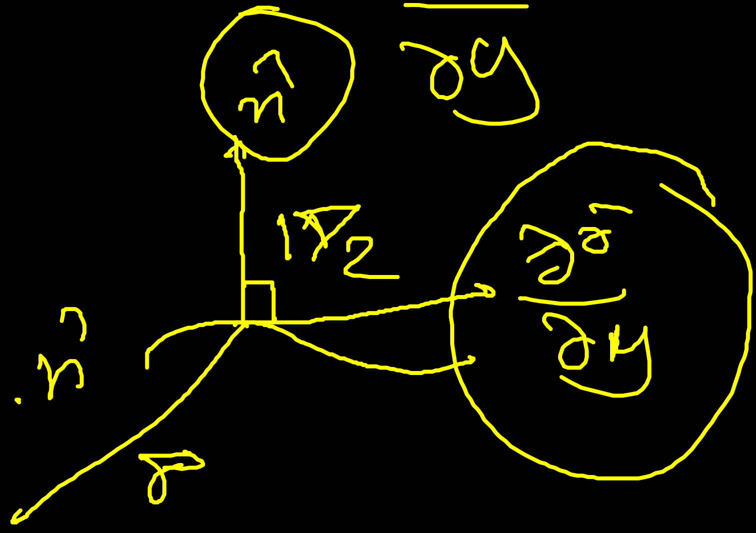
$$= -\frac{\partial z}{\partial y}\hat{k}\cdot\hat{n}$$

S $\hat{n}$

$\vec{r}$

$$\frac{\partial \vec{r}}{\partial y}$$





$$\bar{A}\cdot\bar{B} = AB\cos\theta$$

$$\cancel{\bar{A}\times\bar{B} = AB\sin\theta\,\hat{n}}$$

$$\theta = 90^\circ, \cos\theta = 0$$

$$\hat{A}\cdot\bar{B} = 0$$

$$\iint_S (\nabla \times A_1 i) \cdot \hat{n}\, ds$$

$$= \iint_S \left(\frac{\partial A_1}{\partial z} j - \frac{\partial A_1}{\partial y} k \right) \cdot \hat{n}\, ds$$

$$= \iint_S \left(\frac{\partial A_1}{\partial z} j \cdot \hat{n} - \frac{\partial A_1}{\partial y} k \cdot \hat{n} \right) ds$$

$$= \iint_S \left(-\frac{\partial A_1}{\partial z} \frac{\partial z}{\partial y} - \frac{\partial A_1}{\partial y} \right) k \cdot \hat{n}\, ds$$

$$\iint_S (\nabla \times A_1 \hat{i}) \cdot \hat{n}\, ds$$

$$= -\iint_S \left(\frac{\partial A_1}{\partial y} + \frac{\partial A_1}{\partial z} \frac{\partial z}{\partial y} \right) \hat{k} \cdot \hat{n}\, ds$$

$$A_1 \equiv A_1(x, y, z) = A_1(x, y, f(x, y))$$

$$A_1(x, y, z) = F(x, y) \quad \Big| \quad F(x, y) = A_1(x, y, z)$$

$$\frac{\partial F}{\partial y} = \frac{\partial A_1}{\partial y} + \frac{\partial A_1}{\partial z} \frac{\partial z}{\partial y}$$

$$\iint_S \nabla \times A_1 i \cdot \hat{n}\, ds = -\iint_R \frac{\partial F}{\partial y}\, \underline{\hat{k} \cdot \hat{n}\, ds}$$

$$= -\iint_R \frac{\partial F}{\partial y}\, dx\, dy = \oint_F F\, dx$$

$$\iint_R \quad -\frac{\partial M}{\partial y} \Big) dx\, dy = \oint_C \Big(M dx + \quad \Big)$$

$$N = 0$$

(i) $\iint_S (\vec{\nabla} \times A_1 \hat{i}) \cdot \hat{n}\, ds = \oint_C A_1 dx$

(ii) $\iint_S (\vec{\nabla} \times A_2 \hat{j}) \cdot \hat{n}\, ds = \oint_C A_2 dy$

(iii) $\iint_S (\vec{\nabla} \times A_3 \hat{k}) \cdot \hat{n}\, ds = \oint_C A_3 dz$

(i) + (ii) + (iii) ⇒ $\iint_S (\vec{\nabla} \times \bar{A}) \cdot \hat{n}\, ds = \oint_C \bar{A} \cdot d\bar{r}$

$$A_1(x,y,z) = x^2 y z$$

S: $2x + 3y + 4z = 5$

$$z = \frac{1}{4}(5 - 2x - 3y) = f(x,y)$$

dn

$$A_1(x,y,z) = x^2 y \cdot \frac{1}{4}(5 - 2x - 3y) = F(x,y)$$

dn

$\left(1, \frac{1}{3}, \frac{1}{2}\right)$

$$A_1\left(1, \tfrac{1}{3}, \tfrac{1}{2}\right) = \tfrac{1}{6}$$

$$F(x,y) = \tfrac{1}{6}$$