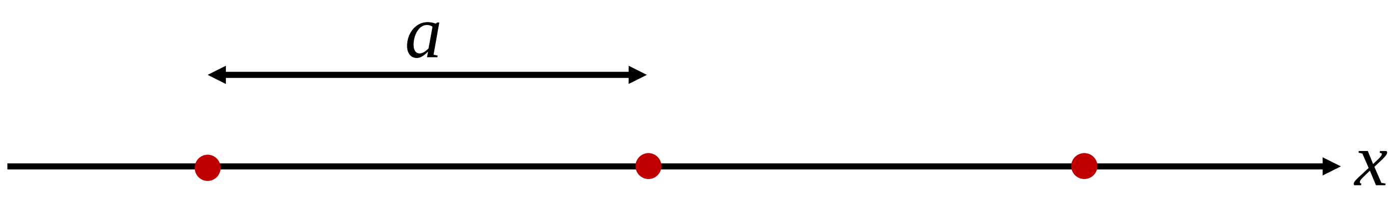
Effective mass of charge carrier

$$\boxed{m^* = \hbar^2 \Big/ \left(\frac{\mathrm{d}^2 E}{\mathrm{d}k^2}\right)}$$

Dr Mohammad Abdur Rashid

Effective mass

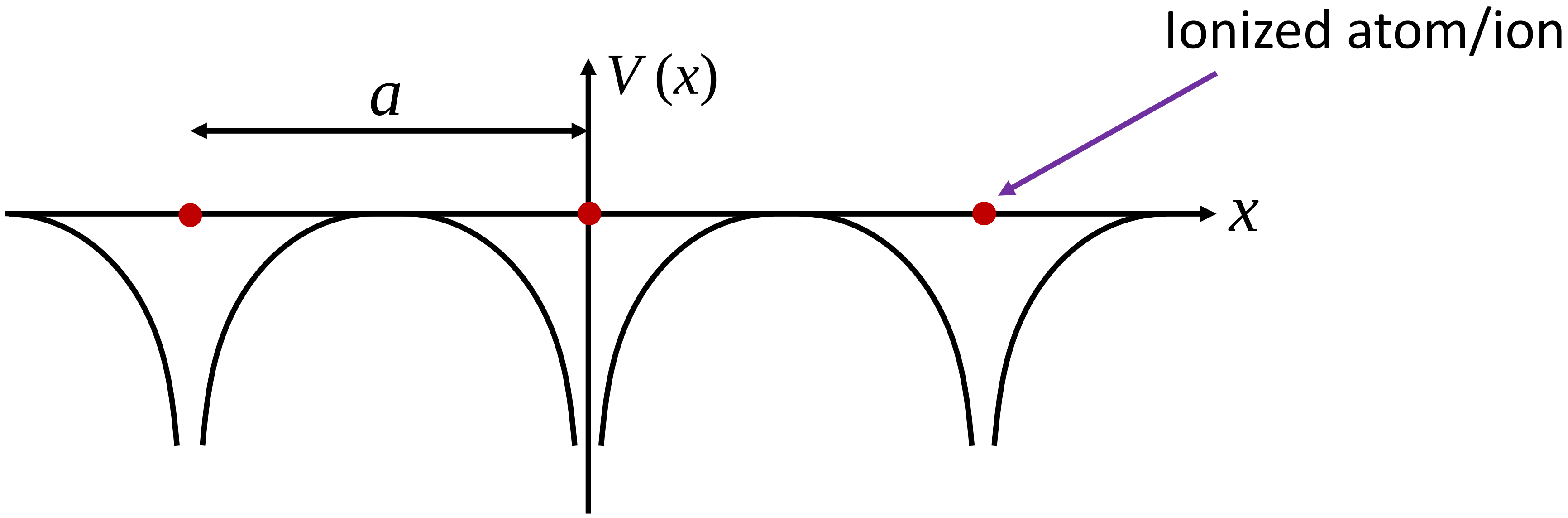
Consider a one-dimensional lattice



Mass of an free electron: $m_0 = 9.1 \times 10^{-31}$ kg

Effective mass

Consider a one-dimensional lattice



Culomb potential of an electron: $V(x) = \dfrac{1}{4\pi\epsilon_0}\dfrac{e(-e)}{|x|}$

What is the Effective Mass

An electron in crystal may behave as if it had a mass different from the free electron mass m_0. There are crystals in which the effective mass of the carriers is much larger or much smaller than m_0. The effective mass may be anisotropic, and it may even be negative. The important point is that the electron in a periodic potential is accelerated relative to the lattice in an applied electric or magnetic field as if its mass is equal to an effective mass.

Effective mass

The de Broglie wavelength λ of a particle of momentum p is determined by the relation

$$\lambda = \frac{h}{p}.$$

Therefore,

$$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$$

$\hbar = h/2\pi$ is the reduced Planck's constant

$k = 2\pi/\lambda$ is the wave vector

$$\boxed{E = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}}$$

$$\boxed{v = \frac{\hbar k}{m^*}}$$

m^* is the effective mass

Effective mass

The kinetic energy of electron: $E = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$

Therefore, $\frac{\mathrm{d}E}{\mathrm{d}k} = \frac{2\hbar^2 k}{2m^*} = \frac{\hbar k}{m^*}\hbar = \frac{p}{m^*}\hbar = v\hbar$

$$\boxed{v = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k}}$$

Effective mass

When an electric field $\mathscr{E}$ is applied to a crystal, electron in the crystal experiences a force $F = -e\mathscr{E}$.

The rate of absorption of energy by the electron: $\dfrac{\mathrm{d}E(k)}{\mathrm{d}t} = -e\mathscr{E} \cdot v$

Time derivative of E: $\dfrac{\mathrm{d}E(k)}{\mathrm{d}t} = \dfrac{\mathrm{d}E}{\mathrm{d}k}\dfrac{\mathrm{d}k}{\mathrm{d}t}$

$$\boxed{v = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k}}$$

Therefore,

$$\boxed{\hbar\frac{\mathrm{d}k}{\mathrm{d}t} = -e\mathscr{E} = F}$$

Effective mass

When an electric field $\mathscr{E}$ is applied to a crystal, the electron undergoes an acceleration.

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}k}\frac{\mathrm{d}k}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k}\right)\cdot\frac{F}{\hbar}$$

$$\boxed{a = \frac{1}{\hbar^2}\frac{\mathrm{d}^2E}{\mathrm{d}k^2}F}$$

$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\mathrm{d}^2E}{\mathrm{d}k^2}}$$

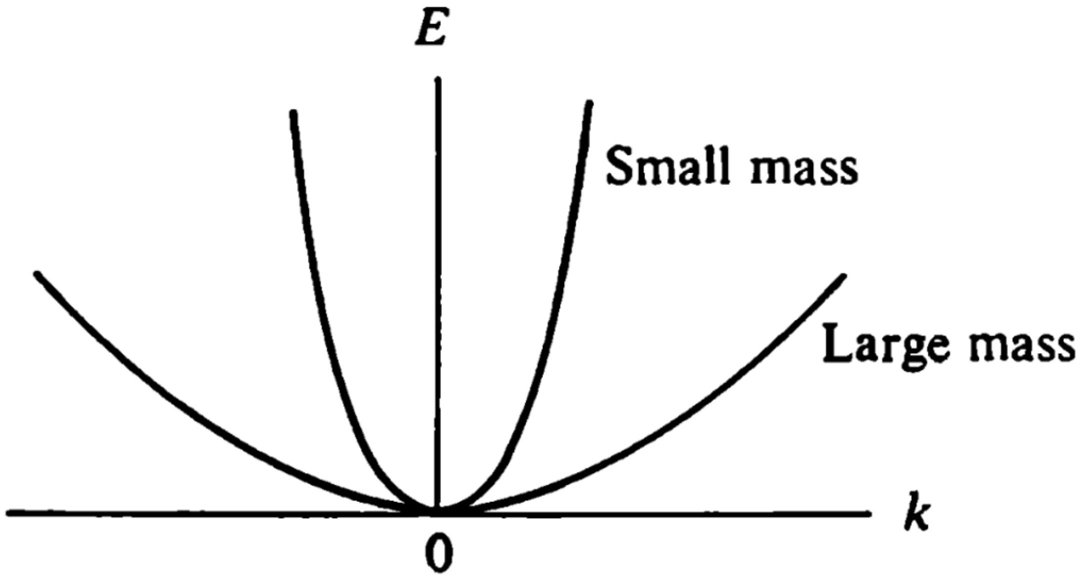
$$\boxed{m^* = \hbar^2 \Big/ \left(\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\right)}$$

m^* is the effective mass

Effective mass

$$\boxed{m^* = \hbar^2 \Big/ \left(\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\right)}$$

The mass m^* is inversely proportional to the curvature of the band; where the curvature is large –that is, $\frac{\mathrm{d}^2E}{\mathrm{d}k^2}$ is large –the mass is small; a small curvature implies a large mass.



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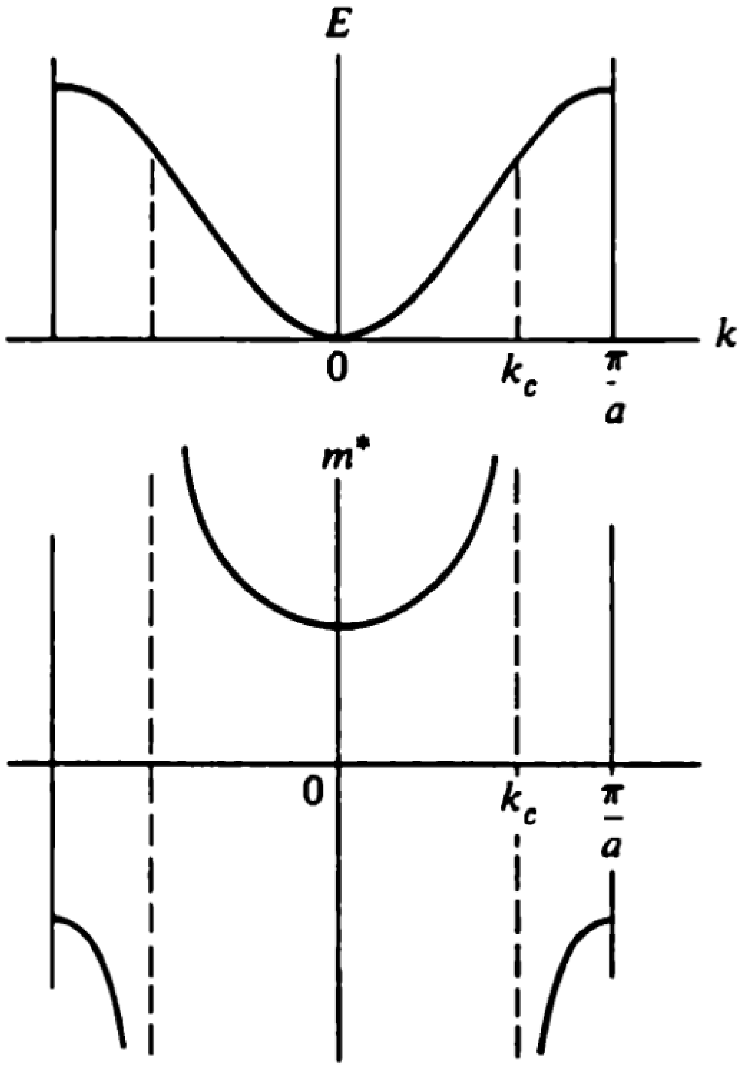
Effective mass

$$E = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

$$m^* = \hbar^2 \Big/ \left(\frac{\mathrm{d}^2 E}{\mathrm{d}k^2}\right)$$

$$\frac{\mathrm{d}^2 E}{\mathrm{d}k^2} = \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{\mathrm{d}E}{\mathrm{d}k}\right)$$



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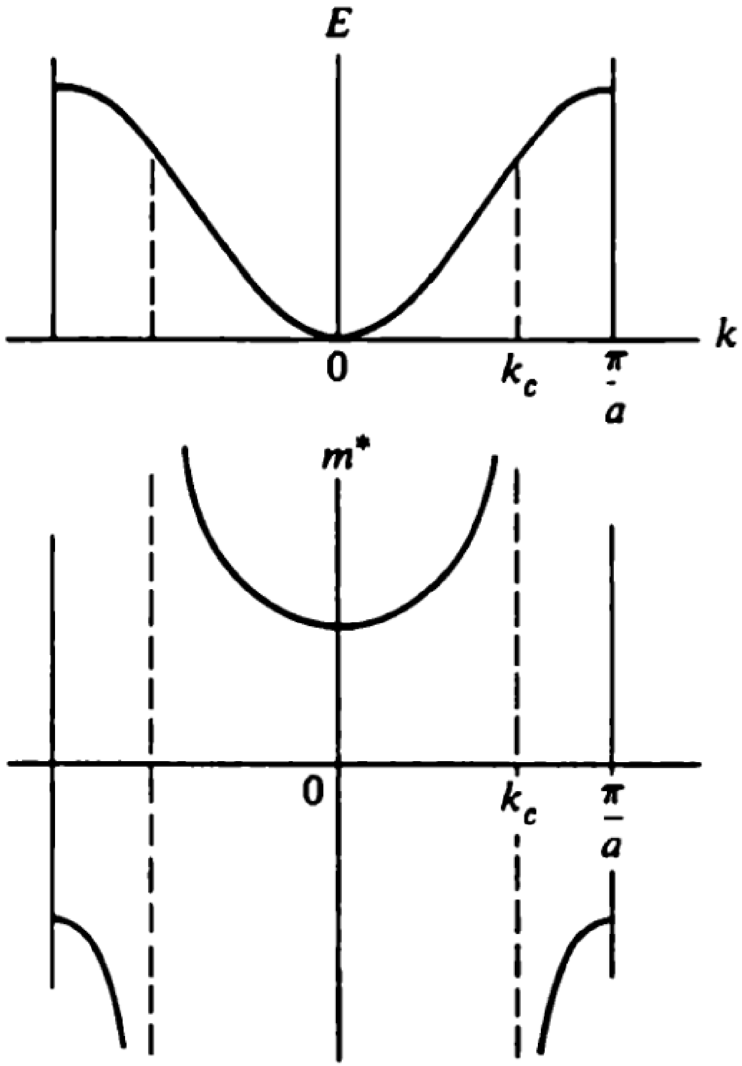
Effective mass

The effective mass m^* is positive near the bottom of the band, where the curvature is positive. But near the top, where the band curvature is negative, the effective mass is also negative.

The fact that the effective mass is different from the free mass is due to the effect of the lattice force on the electron.



Elementary Solid State Physics – Ali Omar

Effective mass

$$\boxed{m^* = \hbar^2 \Big/ \left(\frac{\mathrm{d}^2 E}{\mathrm{d}k^2}\right)}$$

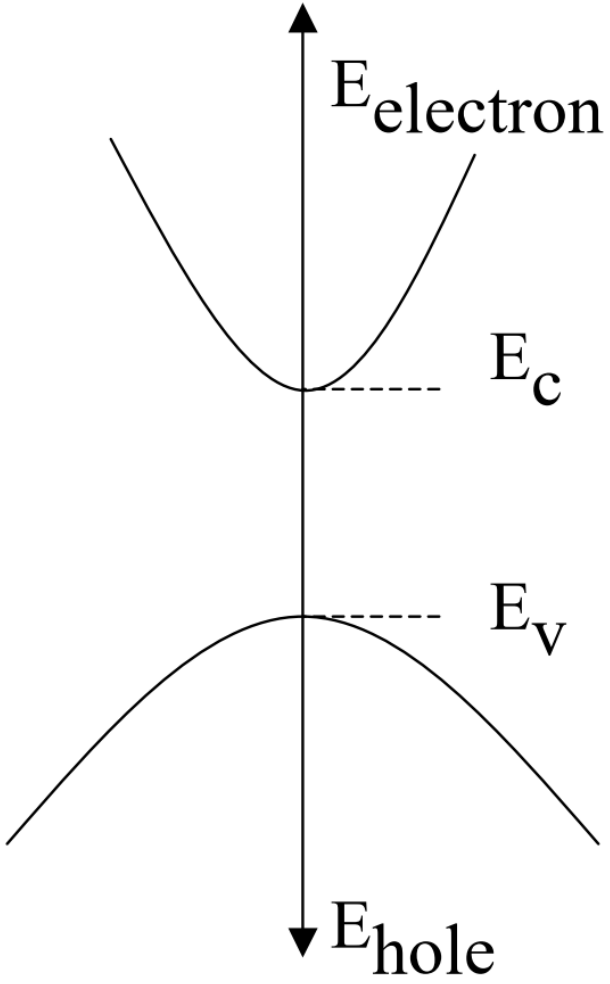
For parabolic bands, the electron will move much like a free particle with m^*, which is related to the curvature of the band. For nonparabolic bands, m^* is not constant and the local slope and curvature of E–k relationship must be used to obtain the velocity and acceleration of the particle with energy E.

Effective mass

The shape of the bottom of the CB and the top of the VB can be approximated by parabolas, which results in constant effective masses.

$$E \approx E_c + \frac{\hbar^2 k^2}{2m_e^*}$$

$$E \approx E_v - \frac{\hbar^2 k^2}{2m_h^*}$$



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