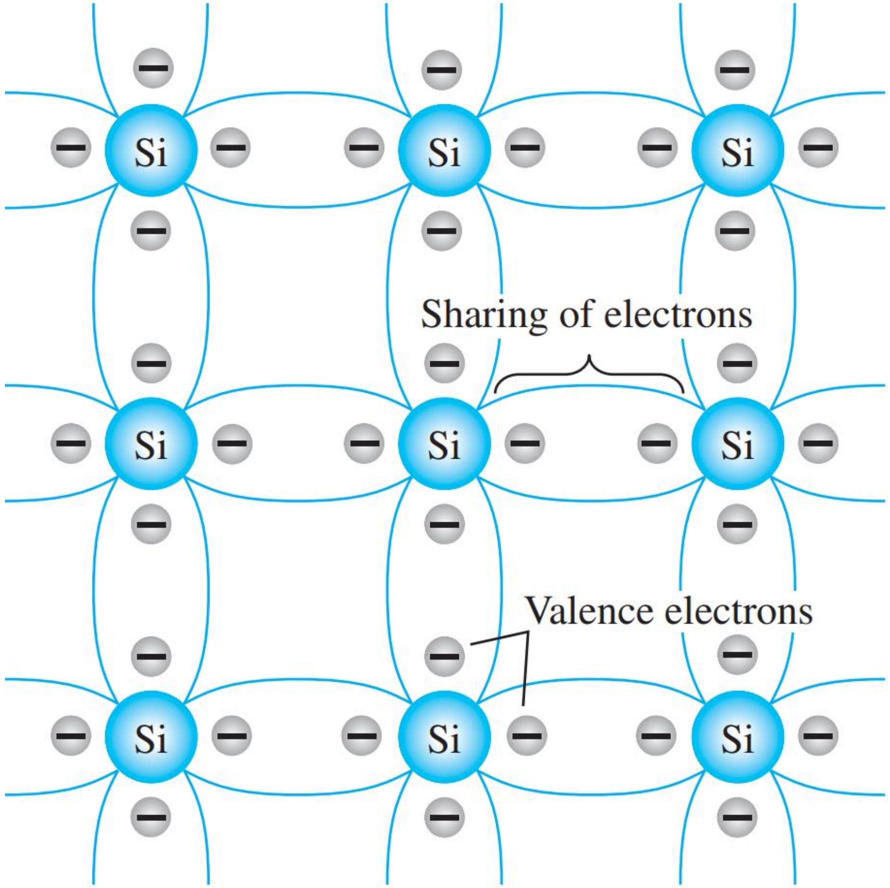
Carrier concentration in intrinsic semiconductor

$$\boxed{n_i = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2k_B T}}$$

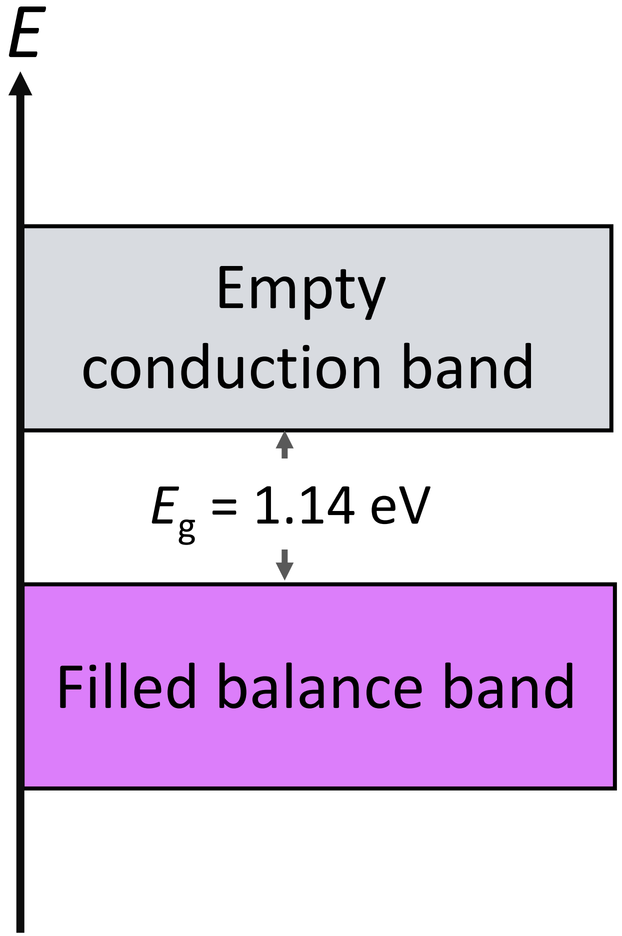
Dr Mohammad Abdur Rashid

Covalent bonding of the silicon atom

At $T = 0$ K

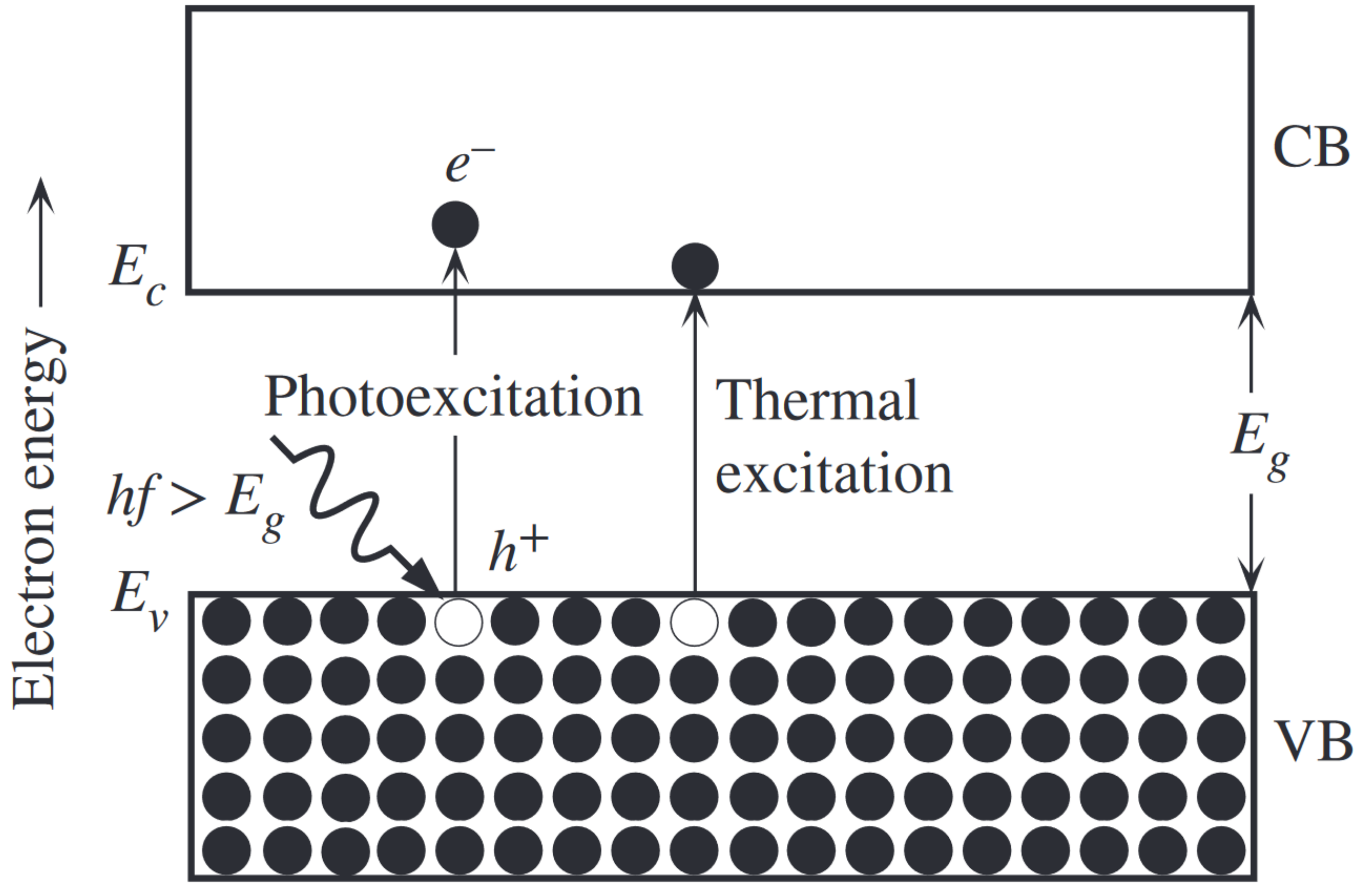


Electronic Devices and Circuit Theory – Boylestad, Nashelsky



Excitation of electrons from VB to CB

At room temperature



Electronic Materials & Devices – Kasap

Density of states

Density of states, $g(E)$:

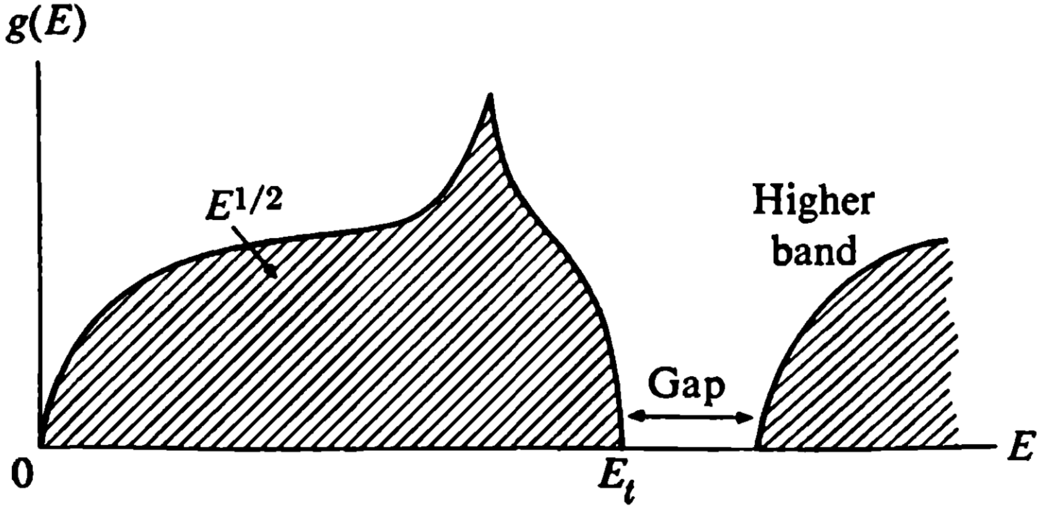
- $g(E)$ is the number of states per unit volume per unit energy range
- $g(E)\,\mathrm{d}E$ is the number of states per unit volume in the energy range $(E, E + \mathrm{d}E)$

$$\boxed{g(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \times \text{volume}}}$$

Density of states

$$g(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2}$$

m^* is the effective mass

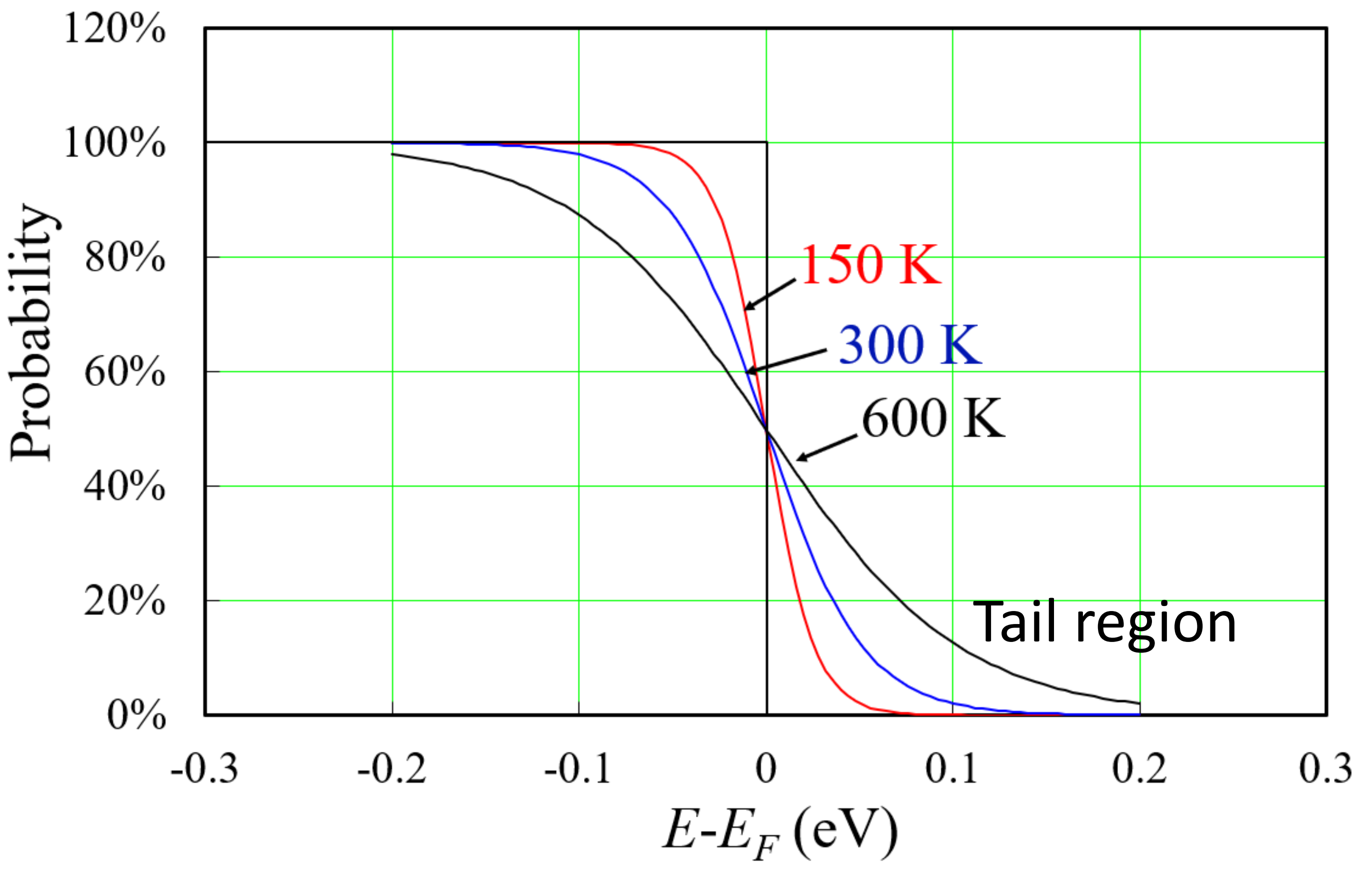


Elementary Solid State Physics – Ali Omar

Mass of an free electron: $m_0 = 9.1 \times 10^{-31}$ kg

Fermi-Dirac (FD) distribution function

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$



Semiconductor Devices – Zeghbroeck

Fermi-Dirac (FD) distribution function

$$\boxed{f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1}}$$

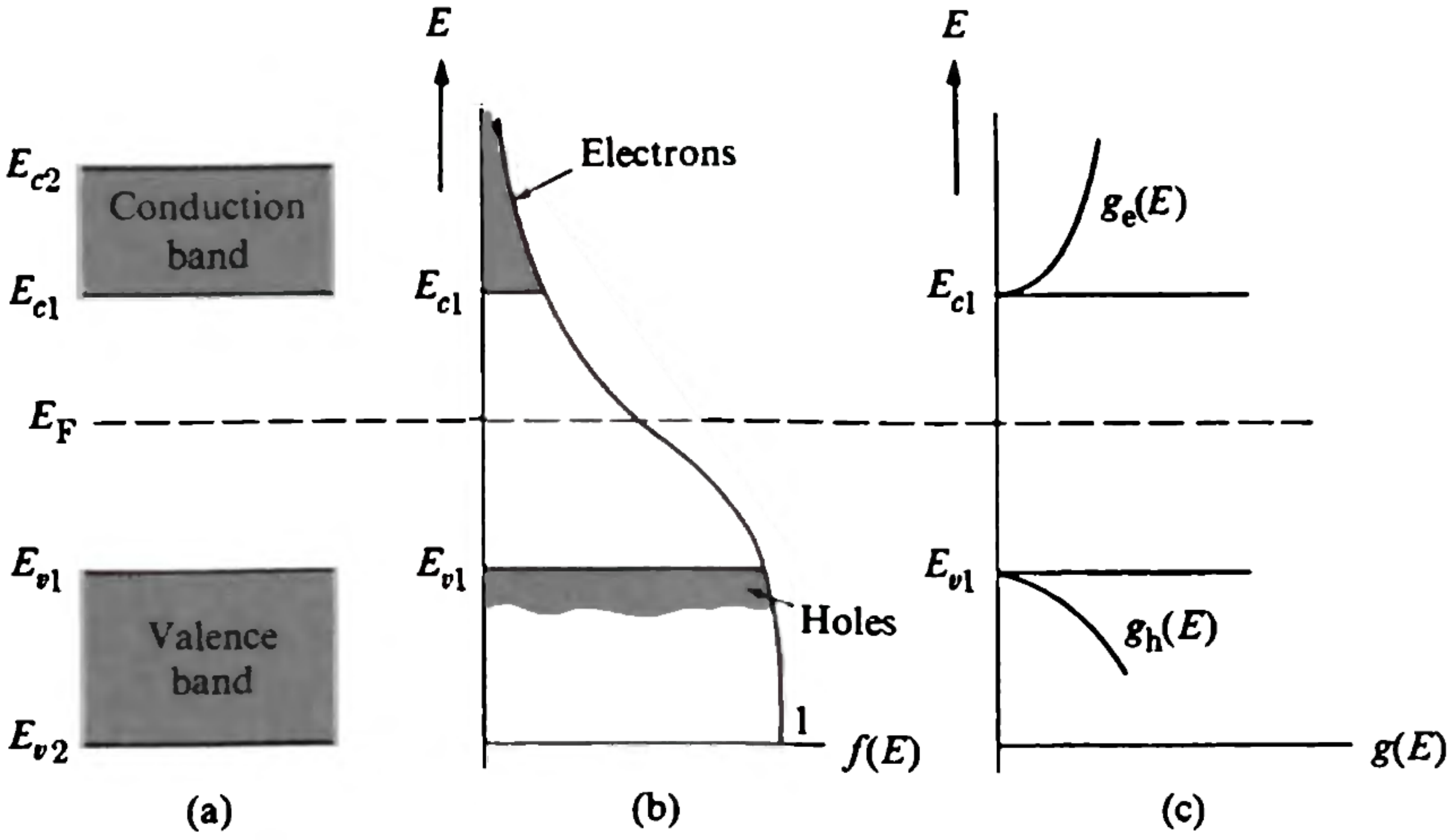
$\boxed{300\ \text{K} \approx 0.026\ \text{eV}}$

In semiconductors it is the tail region of the FD distribution which is of particular interest. In that region the inequality $(E - E_F) >> k_BT$ holds true, and one may therefore neglect the term unity in the denominator. The FD distribution then reduces to

$$\boxed{f(E) = e^{E_F/k_BT}\, e^{-E/k_BT}.}$$

Maxwell-Boltzmann distribution

Density of states for electrons



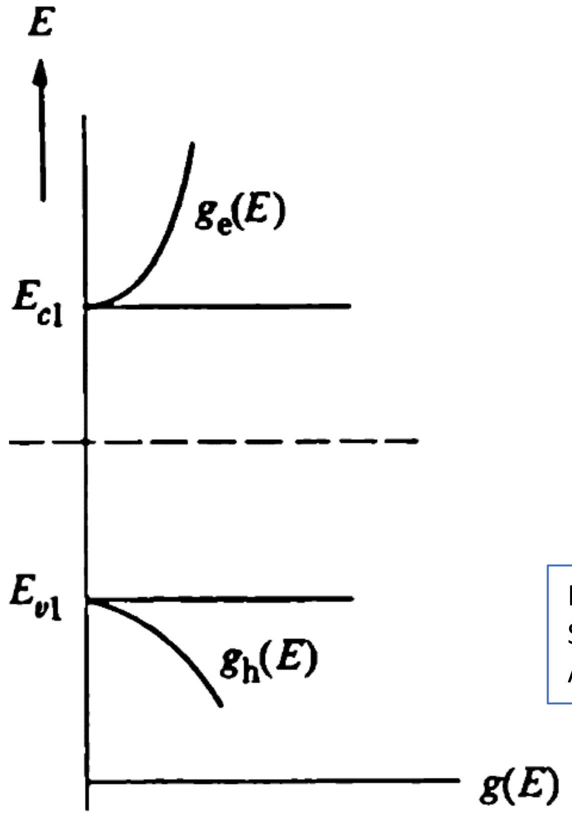
Elementary Solid State Physics – Ali Omar

Density of states for electrons: $g_e(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E - E_g)^{1/2}$

Density of states for electrons

The concentration of electrons throughout the CB is thus given by the integral over the band

$$n = \int_{E_{c1}}^{E_{c2}} f(E) g_e(E) \mathrm{d}E$$



Elementary Solid State Physics – Ali Omar

Density of states for electrons: $\boxed{g_e(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E - E_g)^{1/2}}$

Electron concentration in CB

$$n = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{E_F/k_BT}\int_{E_g}^{\infty}(E-E_g)^{1/2}e^{-E/k_BT}\mathrm{d}E$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{(E_F-E_g)/k_BT}\int_{E_g}^{\infty}(E-E_g)^{1/2}e^{-(E-E_g)/k_BT}\mathrm{d}E$$

$$\boxed{E-E_g=(k_BT)x}$$

$$\boxed{\int_0^{\infty}x^{1/2}e^{-x}\mathrm{d}x=\frac{\pi^{1/2}}{2}}$$

$$\int_{E_g}^{\infty}(E-E_g)^{1/2}e^{-(E-E_g)/k_BT}\mathrm{d}E=(k_BT)^{3/2}\int_0^{\infty}x^{1/2}e^{-x}\mathrm{d}x$$

Electron concentration in CB

$$\boxed{n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_B T}}$$

$$n = N_c e^{(E_F - E_g)/k_B T}$$

$N_c \equiv 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2}$ is the effective density of states of the conduction band.

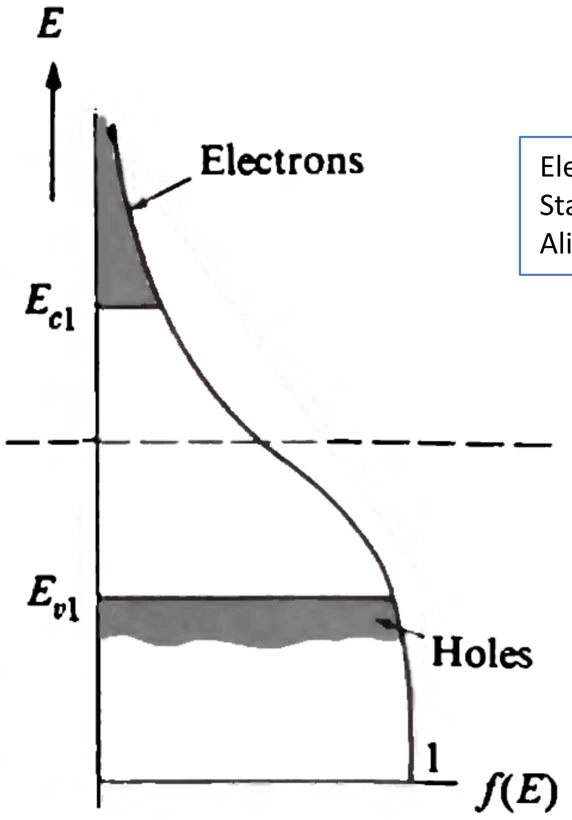
Probability of hole occupation in VB

$$f_h = 1 - f(E)$$

$$\boxed{f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1}}$$

$$f_h = 1 - \frac{1}{e^{(E-E_F)/k_BT} + 1} = \frac{e^{(E-E_F)/k_BT}}{e^{(E-E_F)/k_BT} + 1}$$

$$= \frac{1}{1 + e^{-(E-E_F)/k_BT}} \simeq e^{(E-E_F)/k_BT}$$

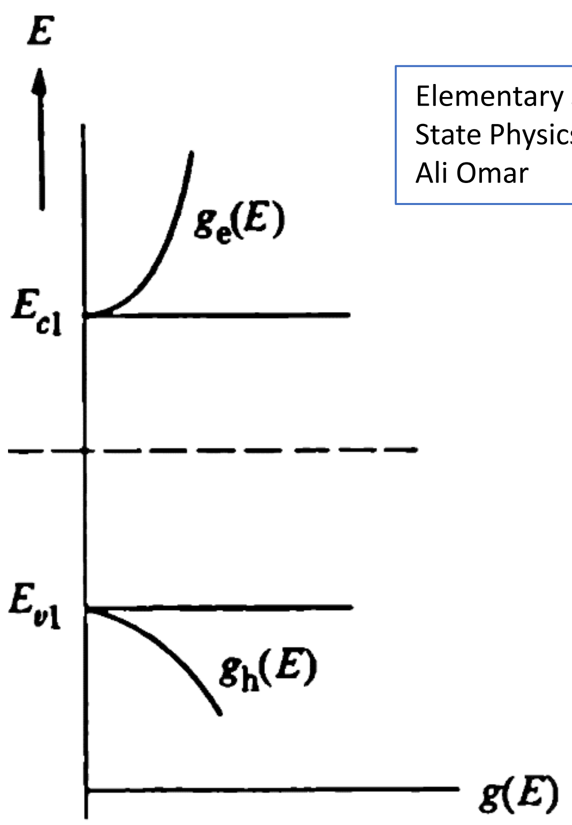


Elementary Solid State Physics – Ali Omar

Hole concentration in VB

The density of states for the holes is

$$g_h = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(-E)^{1/2}$$



Elementary Solid State Physics – Ali Omar

The hole concentration is

$$p = \int_{-\infty}^{0} f_h(E)\, g_h(E) \mathrm{d}E$$

$$\boxed{p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_B T}}$$

Hole concentration in VB

$$\boxed{p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_B T}}$$

$$p = N_v e^{-E_F/k_B T}$$

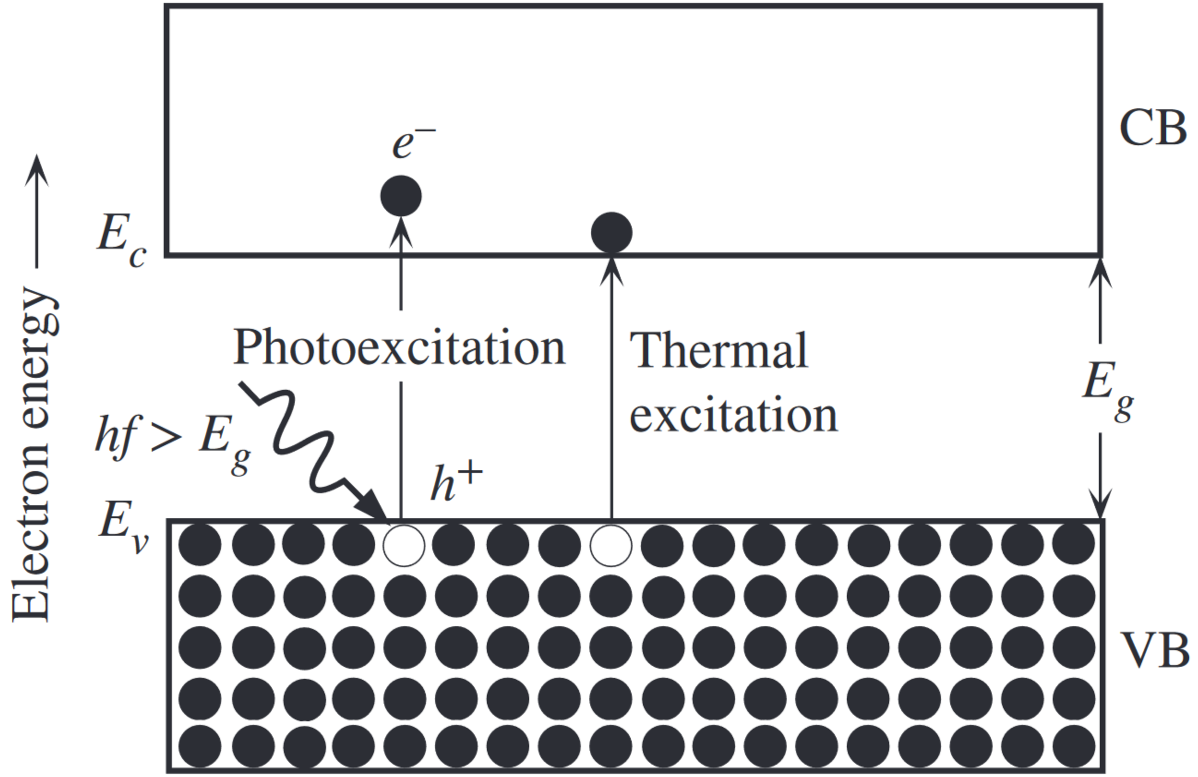
$$N_v \equiv 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} \text{ is the effective density of states of the valance band}$$

Electron and hole concentration

$$\boxed{n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_B T}}$$

$$\boxed{p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_B T}}$$

$$\boxed{n = p}$$



Electronic Materials & Devices – Kasap

Electron and hole concentration

$$\boxed{n = p}$$

$$\boxed{n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(E_F - E_g)/k_B T}}$$

$$\boxed{p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_B T}}$$

$$(m_e)^{3/2}\, e^{(E_F - E_g)/k_B T} = (m_h)^{3/2}\, e^{-E_F/k_B T}$$

$$e^{(2E_F - E_g)/k_B T} = \left(\frac{m_h}{m_e}\right)^{3/2}$$

$$\frac{2E_F - E_g}{k_B T} = \frac{3}{2}\ln\left(\frac{m_h}{m_e}\right)$$

$$\boxed{E_F = \frac{1}{2}E_g + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)}$$

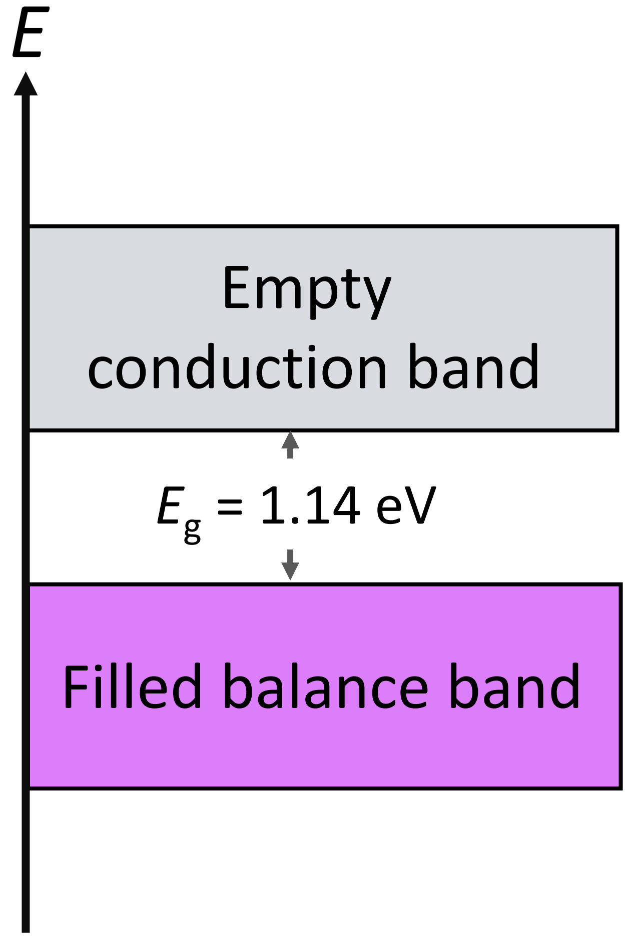
Fermi energy for intrinsic semiconductor

$$\boxed{E_F = \frac{1}{2}E_g + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)}$$

300 K ≈ 0.026 eV

$$k_B T \ll E_g$$

$$\boxed{E_F \approx \frac{1}{2}E_g}$$



Electron and hole concentration

$$\frac{E_F}{k_BT} = \frac{E_g}{2k_BT} + \ln\left(\frac{m_h}{m_e}\right)^{3/4}$$

$$e^{E_F/k_BT} = e^{E_g/2k_BT}\left(\frac{m_h}{m_e}\right)^{3/4}$$

$$\boxed{n = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-E_g/2k_BT}}$$

$$E_g = 1\ \text{eV}, \qquad T = 300\ \text{K}$$

$$m_e = m_h = 9.1\times 10^{-31}\ \text{kg}$$

$$n \simeq 10^{15}\ \text{electrons/cm}^3$$

$$\boxed{n = 2\left(\frac{m_e k_BT}{2\pi\hbar^2}\right)^{3/2} e^{(E_F-E_g)/k_BT}}$$

$$\boxed{p = 2\left(\frac{m_h k_BT}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/k_BT}}$$

$$\boxed{E_F = \frac{1}{2}E_g + \frac{3}{4}k_BT\ln\left(\frac{m_h}{m_e}\right)}$$

Intrinsic carrier concentration

$$n = p = n_i$$

$$\boxed{n_i = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2k_B T}}$$

$$\boxed{n_i = \sqrt{N_c N_v} e^{-E_g/k_B T}}$$