Rectifier

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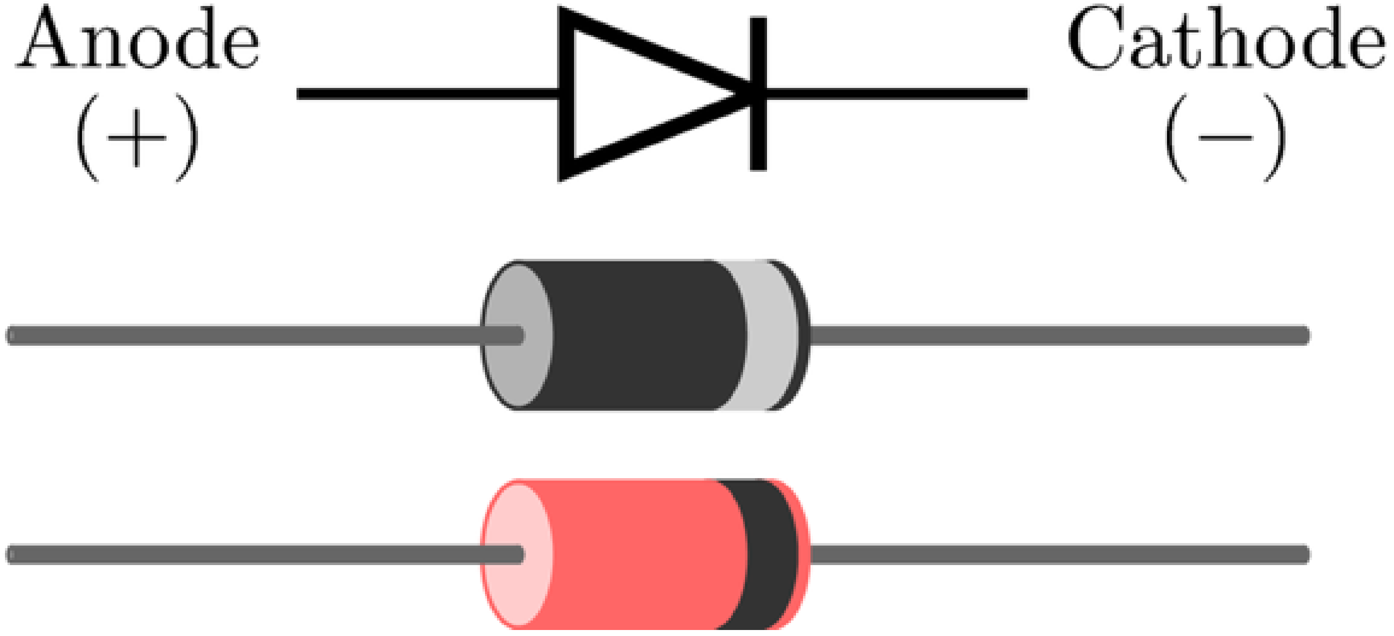
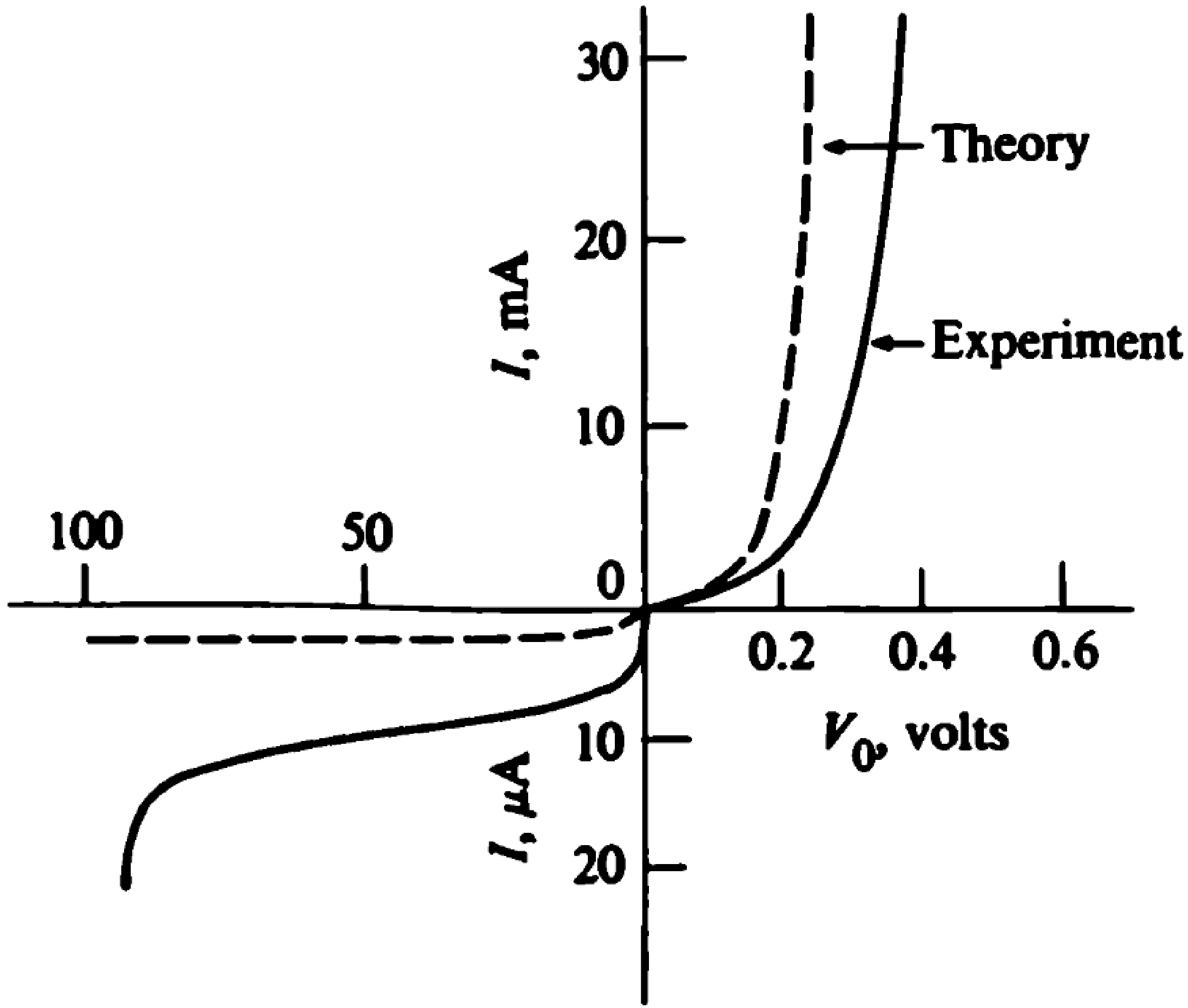
Diode

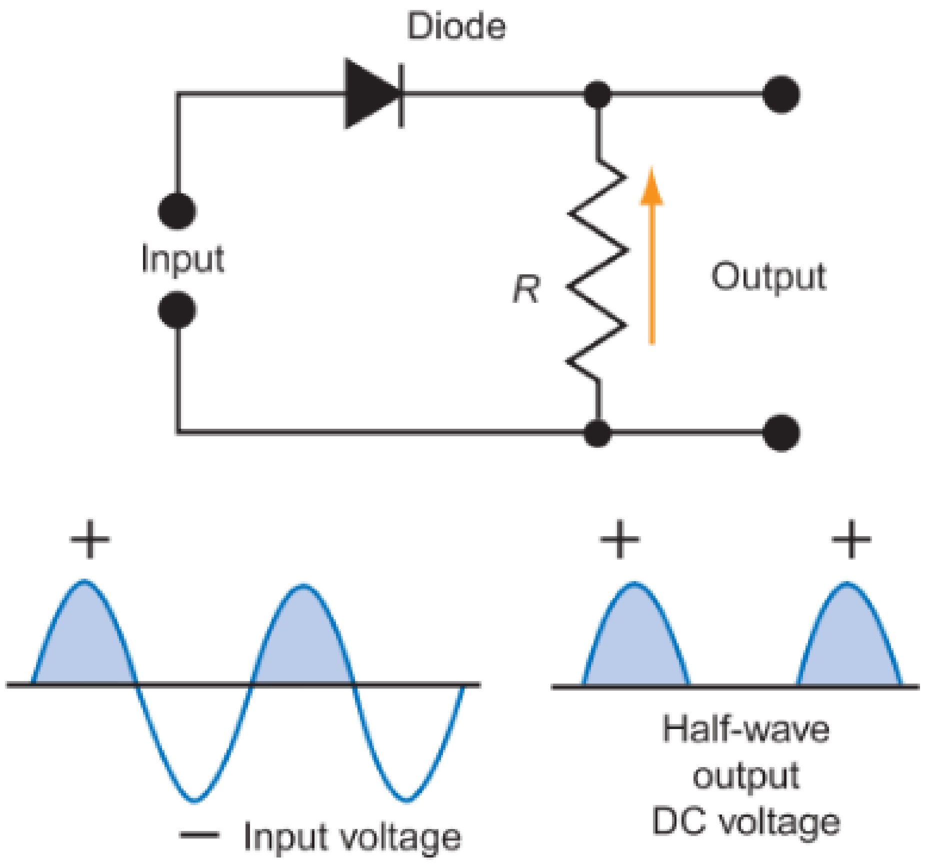


Current versus voltage characteristics

Elementary Solid State Physics – Ali Omar



Half-wave rectifier



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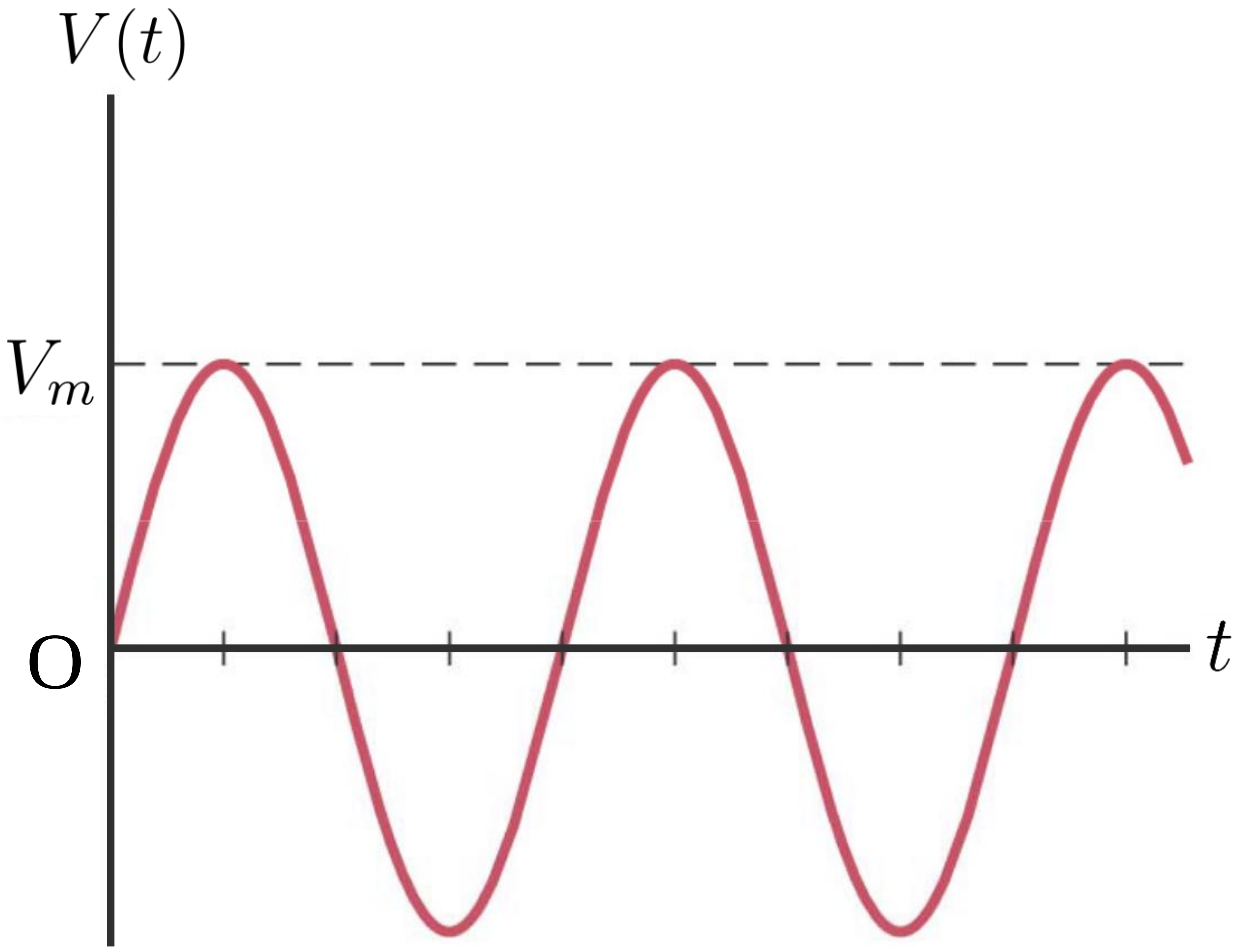
Alternating current (AC)

$$V(t) = V_m \sin(\omega t)$$



Average of $V(t)$ over time T

$$\overline{V} = \frac{1}{T}\int_0^T V(t)\mathrm{d}t$$

$$V(t) = V_m \sin(\omega t)$$

$$\begin{aligned}
\overline{V} &= \frac{V_m}{T}\int_0^T \sin(\omega t)\mathrm{d}t \\
&= \frac{V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^T \\
&= \frac{V_m}{\omega T}\{-\cos(\omega T) + \cos 0\} \\
&= \frac{V_m}{2\pi}\{-\cos(2\pi) + \cos 0\} \\
&= \frac{V_m}{2\pi}(-1+1) \\
&= 0.
\end{aligned}$$

Average of $V(t)$ over time $T/2$

$$V_{\text{avg}} = \frac{1}{T/2}\int_0^{T/2} V(t)\mathrm{d}t$$

$$V(t) = V_m \sin(\omega t)$$

$$\begin{aligned}
V_{\text{avg}} &= \frac{2V_m}{T}\int_0^{T/2} \sin(\omega t)\mathrm{d}t \\
&= \frac{2V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^{T/2} \\
&= \frac{2V_m}{\omega T}\{-\cos(\omega T/2) + \cos 0\} \\
&= \frac{2V_m}{2\pi}\{-\cos(\pi) + \cos 0\} \\
&= \frac{2}{\pi}V_m \\
&\approx 0.637\, V_m.
\end{aligned}$$

The RMS value of *V*(*t*)

The term "RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, is equivalent to a DC voltage that would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor.

RMS is not an "Average" voltage, and its mathematical relationship to peak voltage varies depending on the type of waveform.

$$V_{\text{rms}} = \left[\frac{1}{T} \int_0^T V^2(t) \mathrm{d}t \right]^{1/2}$$

The RMS value of $V(t)$

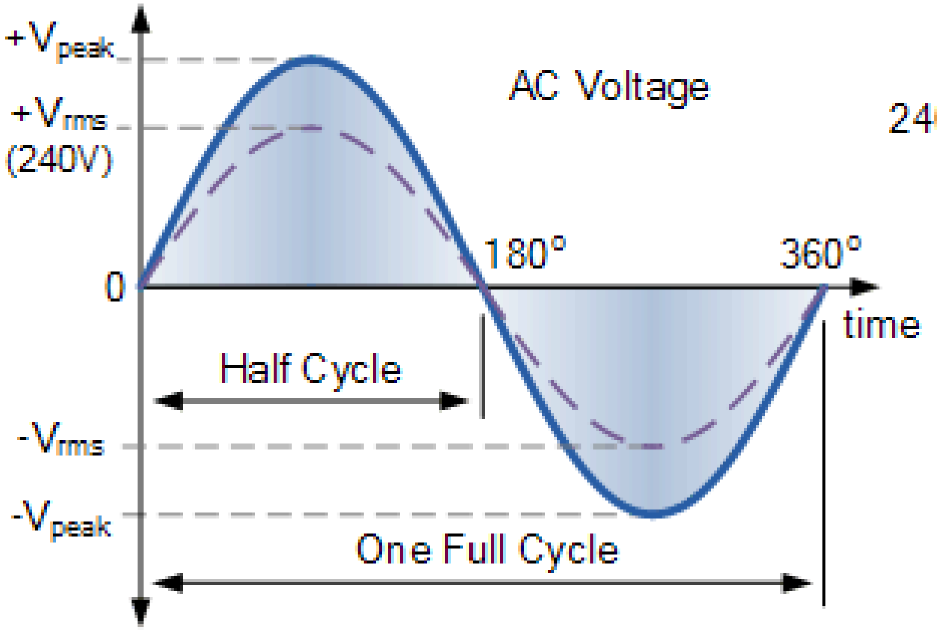
$$
\begin{aligned}
V_{\text{rms}}^2 &= \frac{V_m^2}{T}\int_0^T \sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T 2\sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T \{1-\cos(2\omega t)\}\,\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T \mathrm{d}t - \frac{V_m^2}{T}\int_0^T \cos(2\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\Big[T\Big]_0^T - \frac{V_m^2}{2T}\left[\frac{\sin(2\omega t)}{2\omega}\right]_0^T
\end{aligned}
$$

The RMS value of *V*(*t*)

$$
\begin{aligned}
V_{\text{rms}}^2 &= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(2\omega T) - \sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(4\pi) - \sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}(0 - 0) \\
&= \frac{V_m^2}{2}.
\end{aligned}
$$

$$
V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \approx 0.707\, V_m
$$

The RMS and peak value of AC voltage

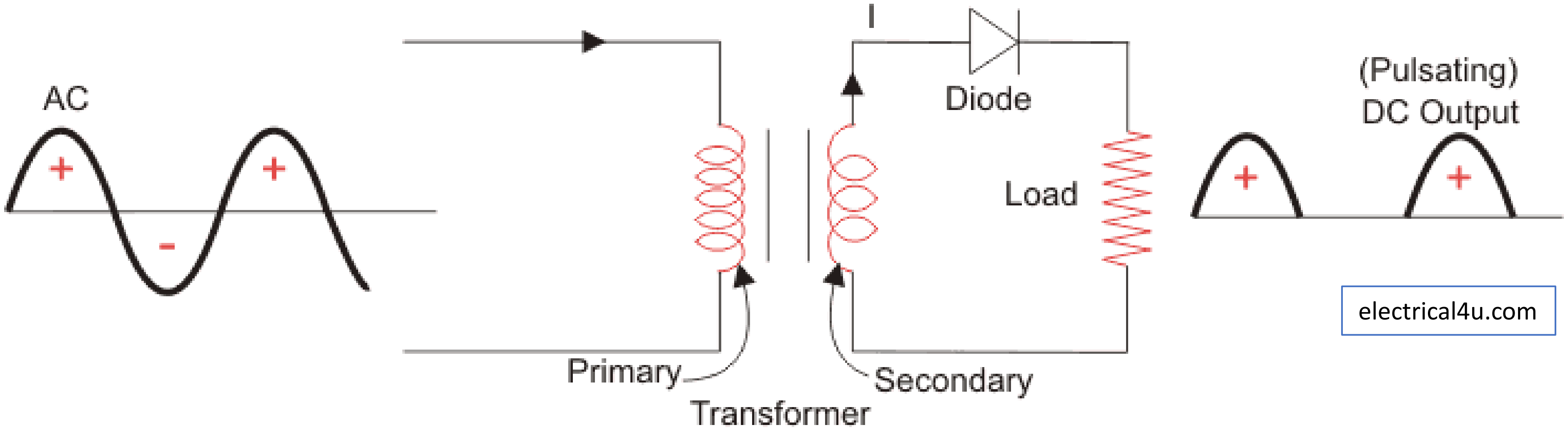


$$V_{\mathrm{rms}} = \frac{V_m}{\sqrt{2}} \approx 0.707\, V_m$$

Important relations to remember

- $V_{\text{rms}} \approx 0.707\times$ peak AC voltage $= 70.7$ % of peak voltage
- Peak AC voltage $\approx 1.414\ \times V_{\text{rms}} = 141.1$ % of V_{rms}
- $V_{\text{avg}} \approx 0.637\times$ peak AC voltage $= 63.7$ % of peak voltage
- $\dfrac{V_{\text{rms}}}{V_{\text{avg}}} = \dfrac{\pi}{2\sqrt{2}} \approx 1.11$

Half wave rectifier (HWR)



$$V_0(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ 0, & T/2 \leq t \leq T \end{cases}$$

Average output voltage of a half wave rectifier

$$
\begin{aligned}
V_{\mathrm{dc}} &= \frac{1}{T}\int_0^T V_0(t)\mathrm{d}t \\
&= \frac{1}{T}\int_0^{T/2} V_m \sin(\omega t)\mathrm{d}t + \frac{1}{T}\int_{T/2}^{T} 0\ \mathrm{d}t \\
&= \frac{V_m}{T}\int_0^{T/2} \sin(\omega t)\mathrm{d}t \\
&= \frac{V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^{T/2} \\
&= \frac{V_m}{\omega T}\{-\cos(\omega T/2) + \cos(0)\} \\
&= \frac{V_m}{\pi}.
\end{aligned}
$$

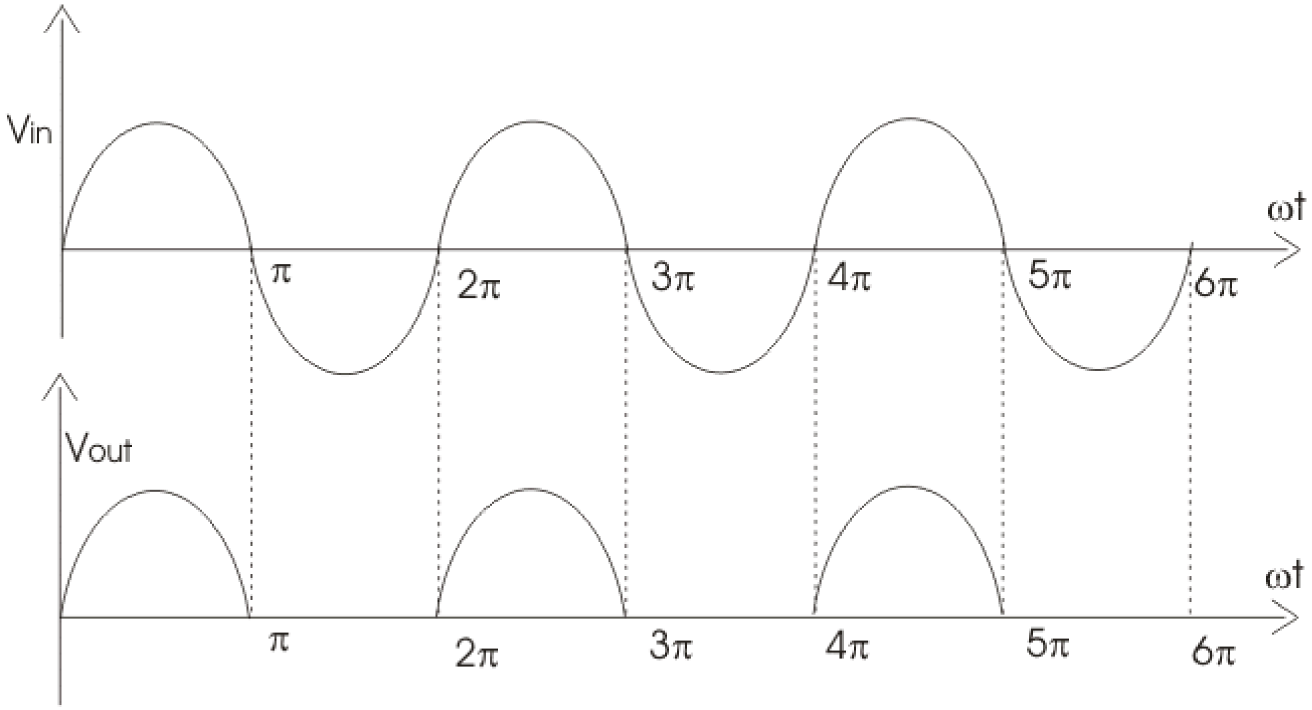
RMS value of the output voltage of a HWR

$$
\begin{aligned}
V_{\mathrm{rms}}^2 &= \frac{1}{T}\int_0^T V_0^2(t)\mathrm{d}t \\
&= \frac{V_m^2}{T}\int_0^{T/2} \sin^2(\omega t)\mathrm{d}t + \frac{V_m^2}{T}\int_{T/2}^{0} 0\ \mathrm{d}t \\
&= \frac{V_m^2}{4}
\end{aligned}
$$

$$
V_0(t) = \begin{cases} V_m \sin(\omega t), & 0 \le t \le T/2 \\ 0, & T/2 \le t \le T \end{cases}
$$

$$
V_{\mathrm{rms}} = \frac{V_m}{2}
$$

Ripple in the output voltage of HWR



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Ripple is the unwanted AC component remaining when converting the AC voltage waveform into a DC waveform. Even though we try out best to remove all AC components, there is still some small amount left on the output side which pulsates the DC waveform. This undesirable AC component is called ripple.

Ripple factor of half wave rectifier

$$\gamma = \frac{\text{RMS value of the AC component}}{\text{value of DC component}} = \frac{V_{\mathrm{r(rms)}}}{V_{\mathrm{dc}}}$$

$$V_0(t) = V_{\mathrm{ac}} + V_{\mathrm{dc}}$$

$$V_{\mathrm{r(rms)}} = \left[\frac{1}{T}\int_0^T V_{\mathrm{ac}}^2 \,\mathrm{d}t\right]^{1/2}$$

$$\begin{aligned} V_{\mathrm{r(rms)}}^2 &= \frac{1}{T}\int_0^T (V_0 - V_{\mathrm{dc}})^2 \,\mathrm{d}t \\ &= \frac{1}{T}\int_0^T (V_0^2 - 2V_0V_{\mathrm{dc}} + V_{\mathrm{dc}}^2) \,\mathrm{d}t \\ &= \frac{1}{T}\int_0^T V_0^2 \,\mathrm{d}t - \frac{2V_{\mathrm{dc}}}{T}\int_0^T V_0 \,\mathrm{d}t + V_{\mathrm{dc}}^2 \\ &= V_{\mathrm{rms}}^2 - 2V_{\mathrm{dc}}^2 + V_{\mathrm{dc}}^2 \\ &= V_{\mathrm{rms}}^2 - V_{\mathrm{dc}}^2. \end{aligned}$$

Ripple factor of half wave rectifier

$$\gamma = \frac{V_{\mathrm{r(rms)}}}{V_{\mathrm{dc}}} = \sqrt{\left(\frac{V_{\mathrm{rms}}}{V_{\mathrm{dc}}}\right)^2 - 1}$$

$$V_{\mathrm{rms}} = \frac{V_m}{2}$$

$$V_{\mathrm{dc}} = \frac{V_m}{\pi}$$

$$\gamma = \sqrt{\left(\frac{V_m}{2} \times \frac{\pi}{V_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} \approx 1.21$$

Efficiency of half wave rectifier

$$\eta = \frac{\text{DC power output}}{\text{AC power input}} = \frac{P_{\text{dc}}}{P_{\text{dc}}}$$

$$P_{\text{dc}} = I_{\text{dc}}^2 R_L = \frac{V_{\text{dc}}^2}{R_L}$$

$$P_{\text{ac}} = I_{\text{rms}}^2 (R_L + r_f) = \frac{V_{\text{rms}}^2}{R_L + r_f}$$

$$\eta_{\text{max}} \approx 0.4053 = 40.53\%$$

$$\begin{aligned}
\eta &= \frac{P_{\text{dc}}}{P_{\text{ac}}} \\
&= \frac{V_{\text{dc}}^2}{R_L} \times \frac{R_L + r_f}{V_{\text{rms}}^2} \\
&= \frac{V_{\text{dc}}^2}{V_{\text{rms}}^2} \times \frac{R_L + r_f}{R_L} \\
&= \left(\frac{V_{\text{dc}}}{V_{\text{rms}}}\right)^2 \times \left(1 + \frac{r_f}{R_L}\right) \\
&= \left(\frac{V_m/\pi}{V_m/2}\right)^2 \times \left(1 + \frac{r_f}{R_L}\right) \\
&\approx 0.4053 \left(1 + \frac{r_f}{R_L}\right)
\end{aligned}$$

Form factor of half wave rectifier

$$\text{f.f.} = \frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} \approx 1.57$$

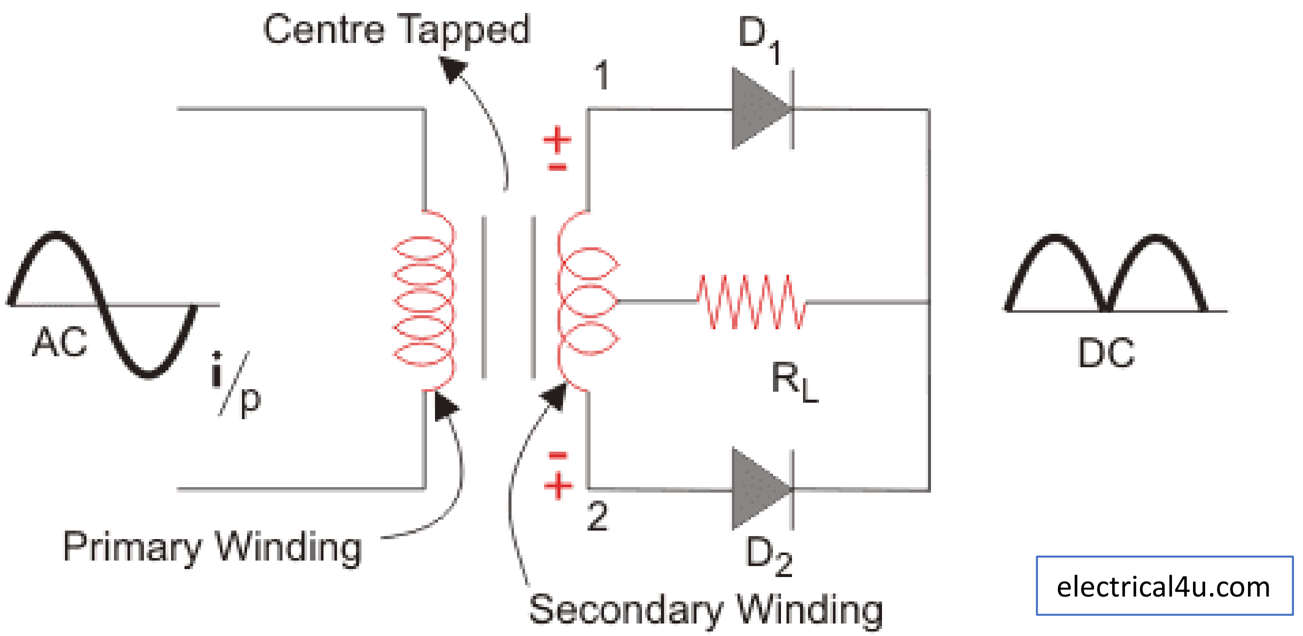
Peak Inverse Voltage (PIV) of half wave rectifier

$$\boxed{\text{PIV} = V_m}$$

Applications of half wave rectifier

- It is used for the detection of amplitude modulated radio signals.
- For the welding purpose, it supplies polarized voltage.
- It is used in many signal demodulation processes.

Full wave rectifier



$$V_0(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ V_m \sin(\omega t - \pi), & T/2 \leq t \leq T \end{cases}$$

Average output voltage of a full wave rectifier

$$V_{\mathrm{dc}} = \frac{1}{T}\int_0^T V_0(t)\mathrm{d}t$$

$$= \frac{1}{T/2}\int_0^{T/2} V_m \sin(\omega t)\mathrm{d}t = \frac{2V_m}{\pi}$$

$$V_0(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ V_m \sin(\omega t - \pi), & T/2 \leq t \leq T \end{cases}$$

RMS value of the output voltage of a FWR

$$V_{\mathrm{rms}} = \left[\frac{1}{T}\int_0^T V_0^2(t)\mathrm{d}t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{T/2}\int_0^{T/2} \sin^2(\omega t)\mathrm{d}t\right]^{1/2} = \frac{V_m}{\sqrt{2}}$$

$$V_0(t) = \begin{cases} V_m \sin(\omega t), & 0 \le t \le T/2 \\ V_m \sin(\omega t - \pi), & T/2 \le t \le T \end{cases}$$

Ripple factor and efficiency of FWR

$$\gamma = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1}$$

$$\approx 0.48$$

$$\eta = \frac{P_{\text{dc}}}{P_{\text{dc}}}$$

$$= \left(\frac{V_{\text{dc}}}{V_{\text{rms}}}\right)^2 \times \left(1 + \frac{r_f}{R_L}\right)$$

$$\approx 0.8106\left(1 + \frac{r_f}{R_L}\right)$$

$$\eta_{\max} \approx 0.8106 = 81.06\%$$

Form factor of full wave rectifier

$$\text{Form factor} = \frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{\pi}{2\sqrt{2}} \approx 1.11$$

Peak Inverse Voltage (PIV) of full wave rectifier

PIV of center tapped full wave rectifier is $2V_m$
and of a bridge rectifiers it is V_m.

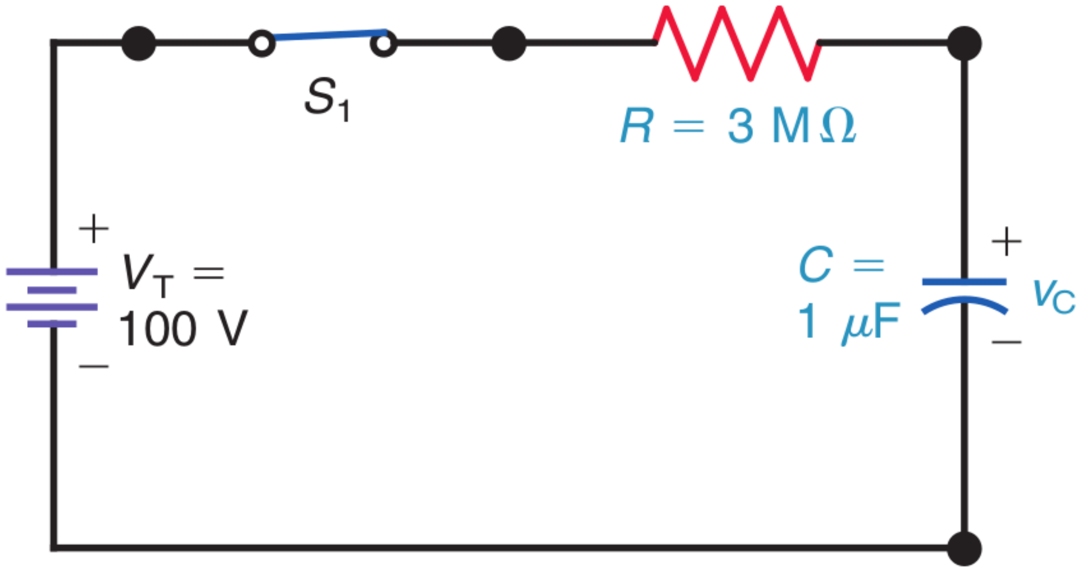
Applications of full wave rectifier

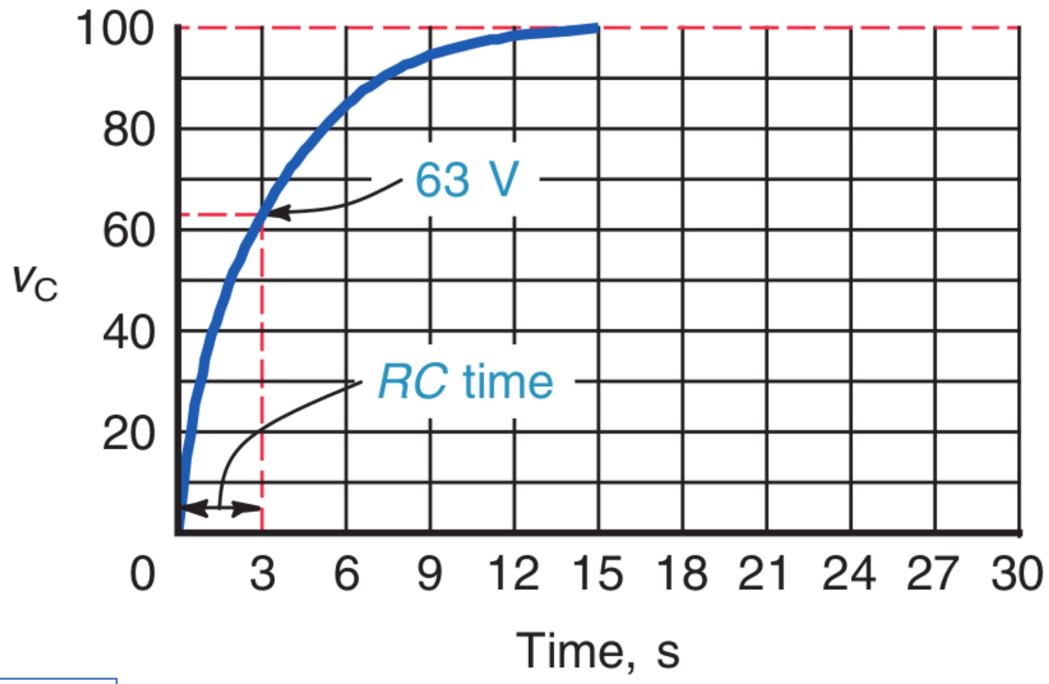
- It can be used to detect the amplitude of modulated radio signal.
- It can be used to supply polarized voltage in welding.
- The Bridge Rectifier circuits are widely used in power supply for various appliances, as they are capable of converting the High AC voltage into Low DC voltage.

A comparison of different parameters

Parameters	Half wave rectifier	Full wave rectifier
Number of diodes	1	2 or 4
Maximum efficiency	40.53%	81.06 %
Peak inverse voltage	V_m	V_m or $2V_m$
Average voltage no load	V_m/π	$2V_m/\pi$
V_{rms} no load	$V_m/2$	$V_m/\sqrt{2}$
Ripple factor	1.21	0.48
Form factor	1.57	1.11
Output frequency	f	$2f$

RC circuit





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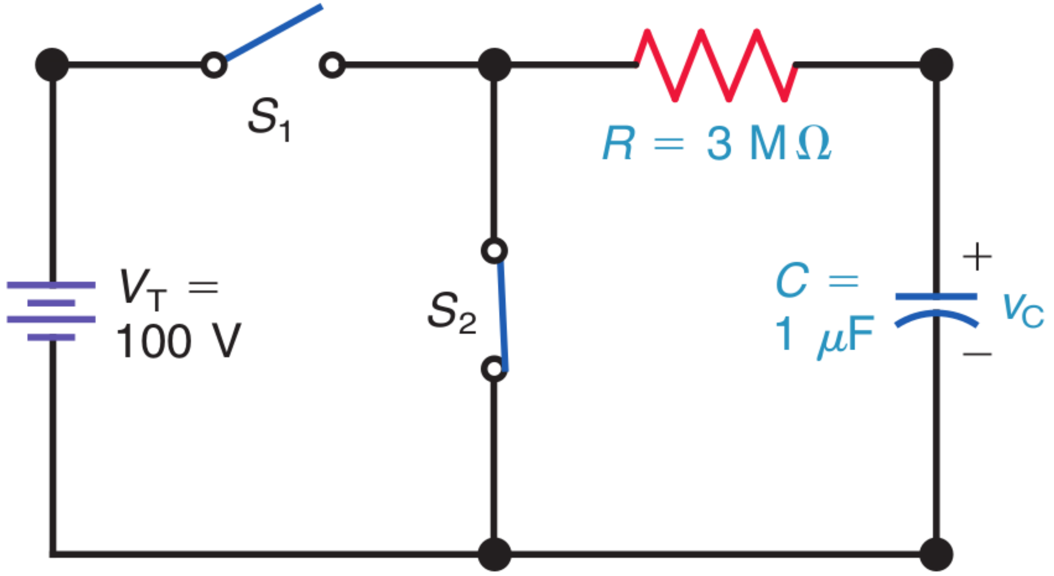
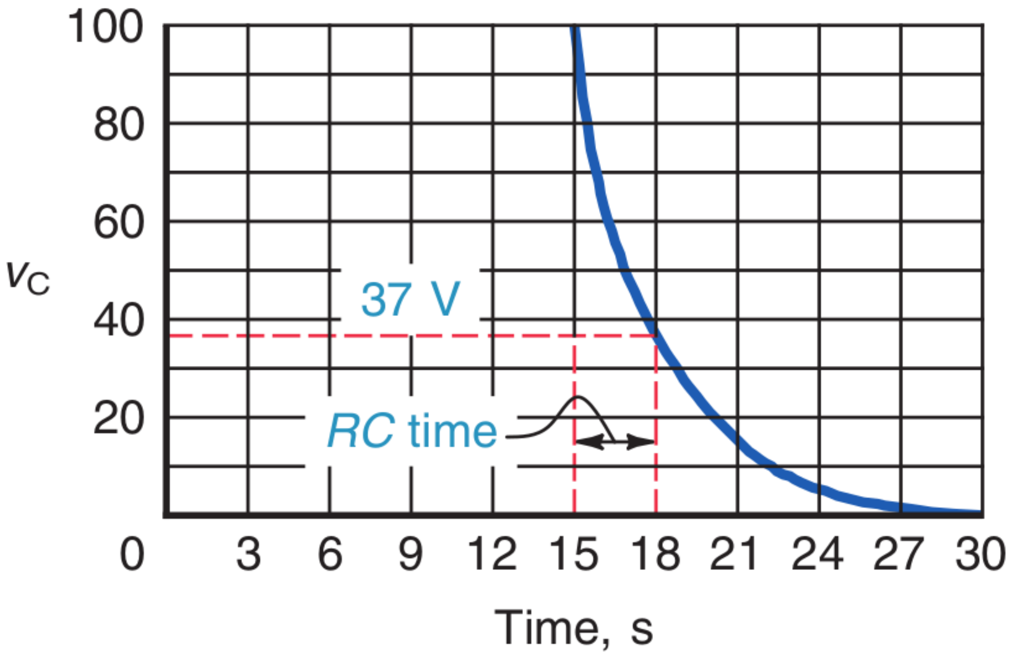
RC circuit





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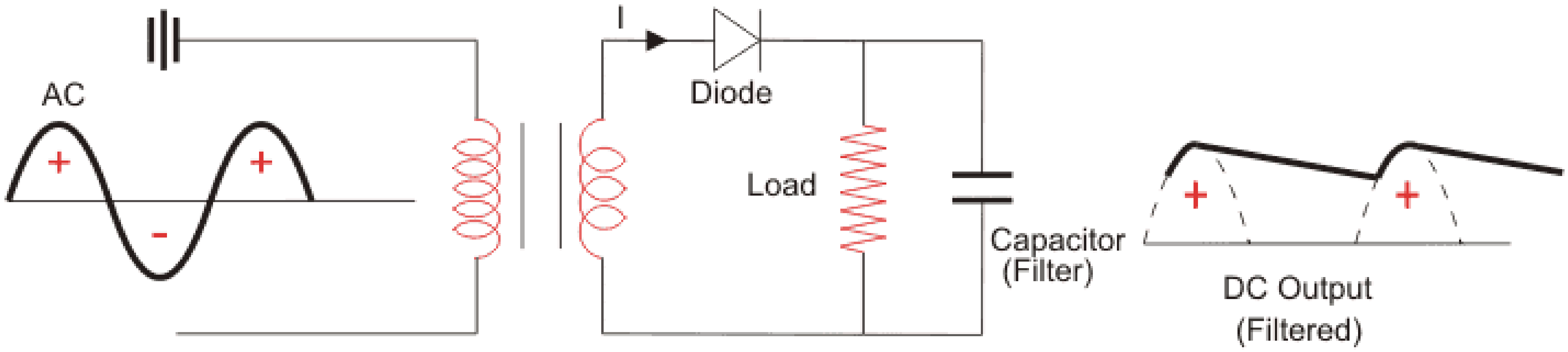
Half wave rectifier with capacitor filter



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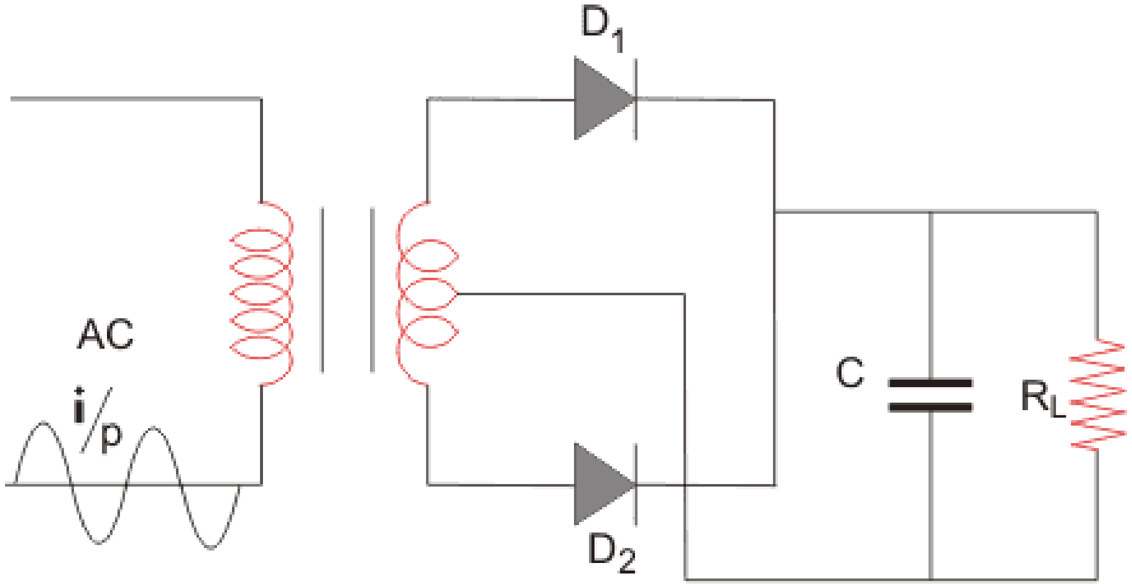
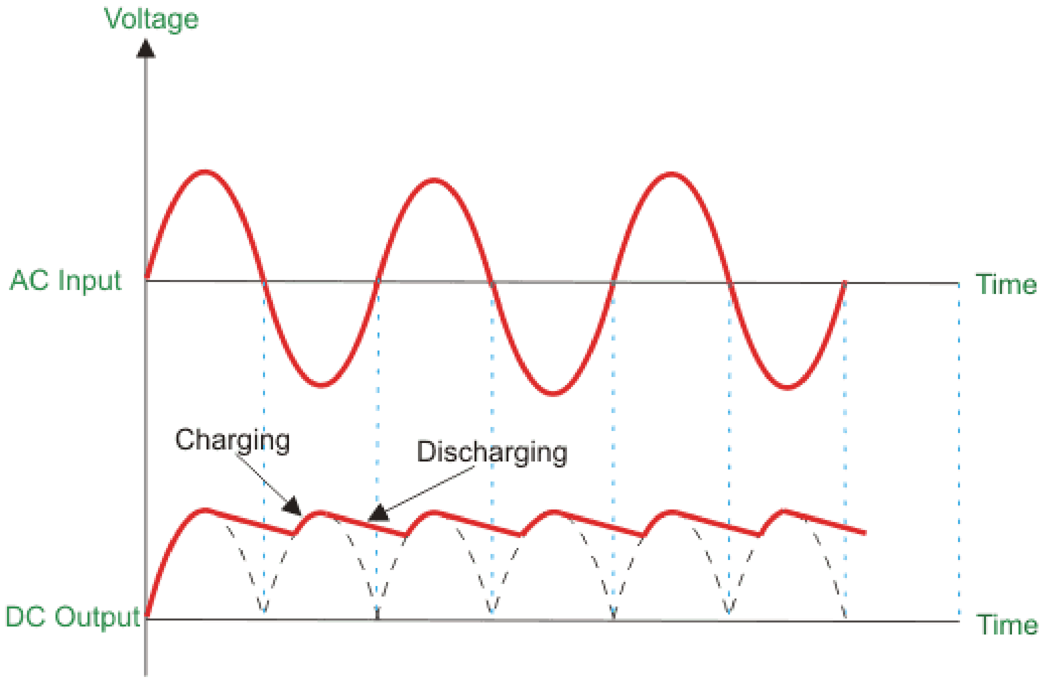
Full wave rectifier with capacitor filter





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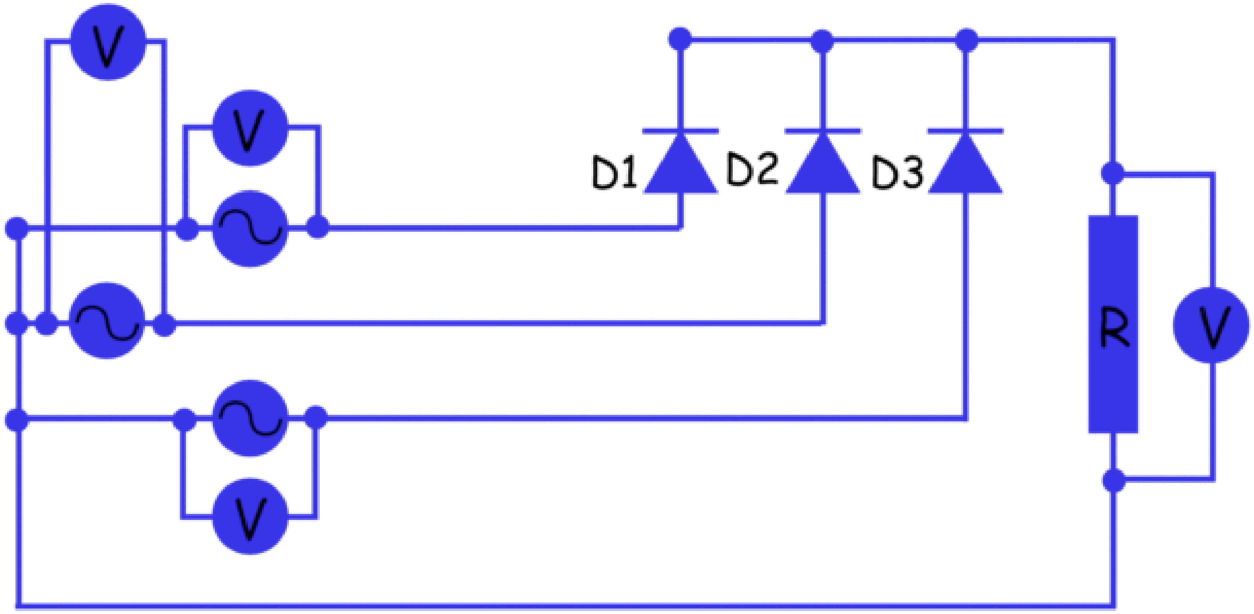
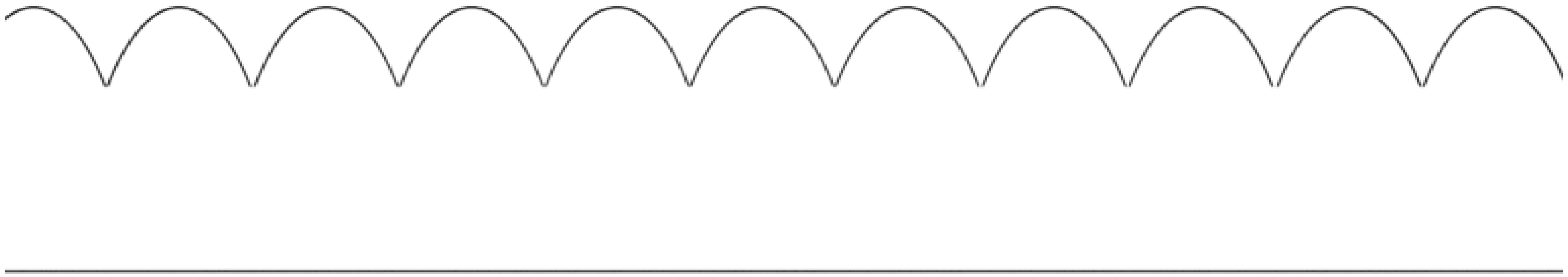
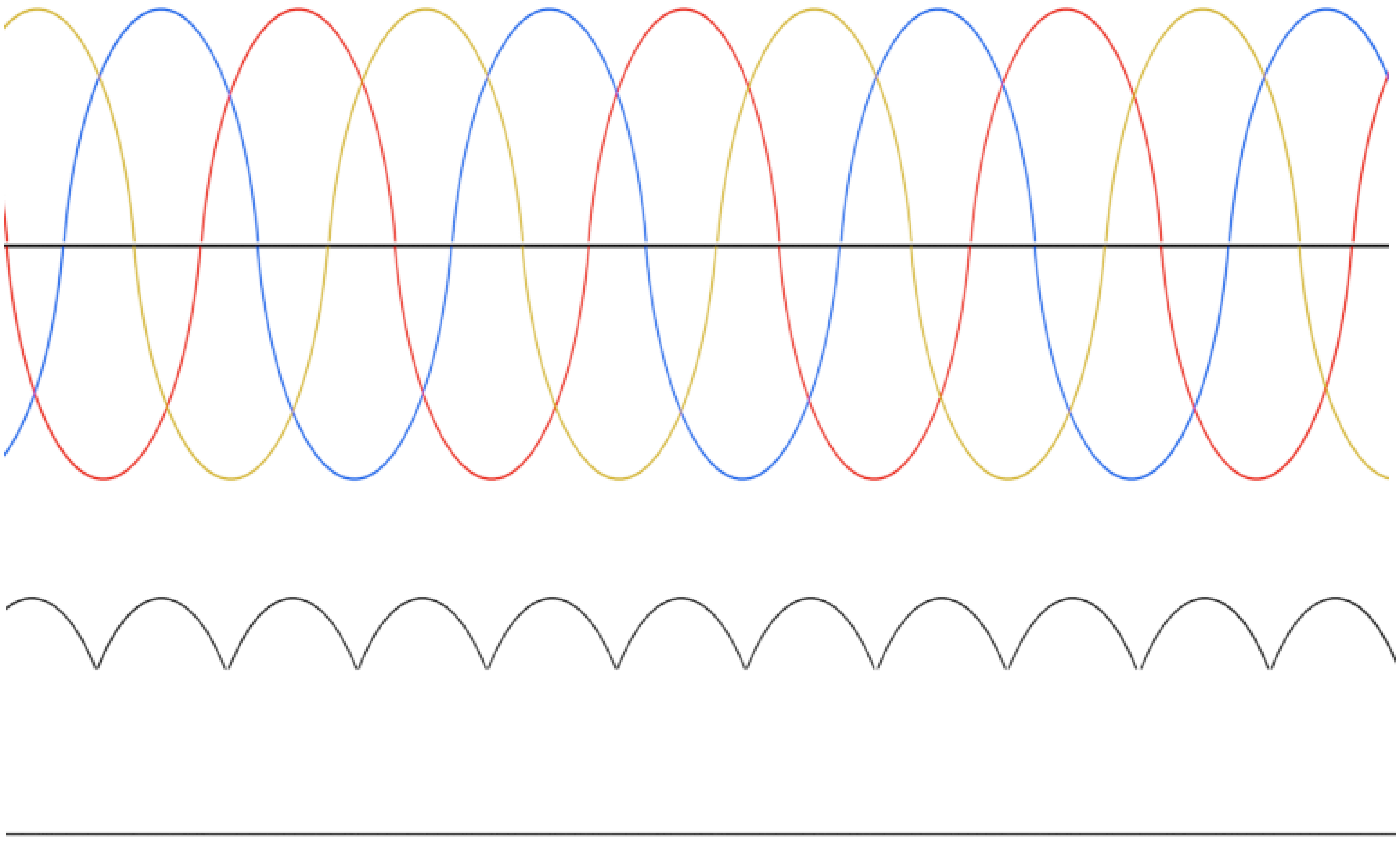
Three phase half wave rectifier





Three phase half wave rectifier



Readings

Grob’s Basic Electronics

– Mitchel E. Schultz

Chapter 27: Diodes and Diode Applications