Dielectric Materials and Insulation

Session: 2022-2023
Course code: PHY 3101
Course title: Electrical Engineering Materials
Bachelor of Science in Electrical and Electronic Engineering
Jashore University of Science and Technology

Content

Dielectric properties of materials, Dielectric constant, polarization- electronic, ionic and orientational; internal field, Clausius-Mosotti equation, spontaneous polarization, frequency dependence of dielectric constant, dielectric loss and piezoelectricity.

References

Principles of electronic materials and devices – S. O. Kasap
Electrical Engineering Materials – A. J. Dekker

CHAPTER

7

Dielectric Materials and Insulation

The familiar parallel plate capacitor equation with free space as an insulator is given by

$$C = \frac{\varepsilon_o A}{d}$$

where ε_o is the absolute permittivity, A is the plate area, and d is the separation between the plates. If there is a material medium between the plates, then the capacitance, the charge storage ability per unit voltage, increases by a factor of ε_r, where ε_r is called the **dielectric constant** of the medium or its **relative permittivity.** The increase in the capacitance is due to the **polarization** of the medium in which positive and negative charges are displaced with respect to their equilibrium positions. The opposite surfaces of the dielectric medium acquire opposite surface charge densities that are related to the amount of polarization in the material. An important concept in dielectric theory is that of an **electric dipole moment** p, which is a measure of the electrostatic effects of a pair of opposite charges $+Q$ and $-Q$ separated by a finite distance a, and so is defined by

$$p = Qa$$

Although the net charge is zero, this entity still gives rise to an electric field in space and also interacts with an electric field from other sources. The relative permittivity is a material property that is frequency dependent. Some capacitors are designed to work at low frequencies, whereas others have a wide frequency range. Furthermore, even though they are regarded as energy storage devices, all practical capacitors exhibit some losses when used in an electric circuit. These losses are no different than I^2R losses in a resistor carrying a current. The power dissipation in a practical capacitor depends on the frequency, and for some applications it can be an important factor. A defining property of a dielectric medium is not only its ability to increase capacitance but also, and equally important, its insulating behavior or low conductivity so that the charges are not conducted from one plate of the capacitor to the other through the dielectric. Dielectric materials often serve to insulate current-carrying conductors or conductors at different voltages. Why can we not simply use air as insulation between high-voltage conductors? When the electric field inside an insulator exceeds a critical field called the **dielectric strength,** the medium suffers dielectric breakdown and a large discharge current flows through the dielectric. Some 40 percent of utility generator failures are linked to insulation failures in the generator. Dielectric breakdown is probably one of the oldest electrical engineering problems and that which has been most widely studied and never fully explained.

7.1 MATTER POLARIZATION AND RELATIVE PERMITTIVITY

7.1.1 RELATIVE PERMITTIVITY: DEFINITION

We first consider a parallel plate capacitor with vacuum as the dielectric medium between the plates, as shown in Figure 7.1a. The plates are connected to a constant voltage supply V. Let Q_o be the charge on the plates. This charge can be easily measured. The capacitance C_o of the parallel plate capacitor in free space, as in Figure 7.1a, is defined by

Definition of capacitance

$$C_o = \frac{Q_o}{V} \tag{7.1}$$

The electric field, directed from high to low potential, is defined by the gradient of the potential $\mathcal{E} = -dV/dx$. Thus, the electric field $\mathcal{E}$ between the plates is just V/d where d is the separation of the plates.

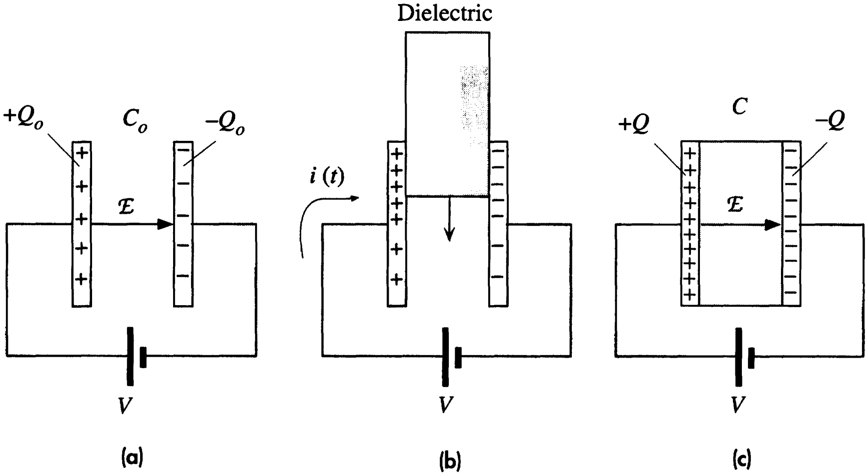


Figure 7.1

(a) Parallel plate capacitor with free space between the plates.

(b) As a slab of insulating material is inserted between the plates, there is an external current flow indicating that more charge is stored on the plates.

(c) The capacitance has been increased due to the insertion of a medium between the plates.

Consider now what happens when a dielectric slab (a slab of any nonconducting material) is inserted into this parallel plate capacitor, as shown in Figure 7.1b and c with V kept the same. During the insertion of the dielectric slab, there is an external current flow that indicates that there is additional charge being stored on the plates. The charge on the electrodes increases from Q_o to Q. We can easily measure the extra charge $Q - Q_o$ flowing from the battery to the plates by integrating the observed current in the circuit during the process of insertion, as shown in Figure 7.1b. Because there is now a greater amount of charge stored on the plates, the capacitance of the system in Figure 7.1c is larger than that in Figure 7.1a by the ratio Q to Q_o. The **relative permittivity** (or the **dielectric constant**) ε_r is defined to reflect this increase in the capacitance or the charge storage ability by virtue of having a dielectric medium. If C is the capacitance with the dielectric medium as in Figure 7.1c, then by definition

$$\varepsilon_r = \frac{Q}{Q_o} = \frac{C}{C_o} \qquad [7.2]$$

Definition of relative permittivity

The increase in the stored charge is due to the polarization of the dielectric by the applied field, as explained below. It is important to remember that when the dielectric medium is inserted, the electric field remains unchanged, provided that the insulator fills the whole space between the plates as shown in Figure 7.1c. The voltage V remains the same and therefore so does the gradient V/d, which means that $\mathcal{E}$ remains constant.

7.1.2 DIPOLE MOMENT AND ELECTRONIC POLARIZATION

An electrical dipole moment is simply a separation between a negative and positive charge of equal magnitude Q as shown in Figure 7.2. If **a** is the vector from the negative to the positive charge, the **electric dipole moment** is defined as a vector by

$$\mathbf{p} = Q\mathbf{a} \qquad [7.3]$$

Definition of dipole moment

The region that contains the $+Q$ and $-Q$ charges has zero net charge. Unless the two charge centers coincide, this region will nonetheless, by virtue of the definition in Equation 7.3, contain a dipole moment.

The net charge within a neutral atom is zero. Furthermore, on average, the center of negative charge of the electrons coincides with the positive nuclear charge, which means that the atom has no net dipole moment, as indicated in Figure 7.3a. However, when this atom is placed in an external electric field, it will develop an induced dipole moment. The electrons, being much lighter than the positive nucleus, become easily displaced by the field, which results in the separation of the negative charge center

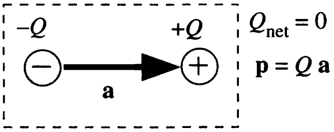


Figure 7.2 The definition of electric dipole moment.

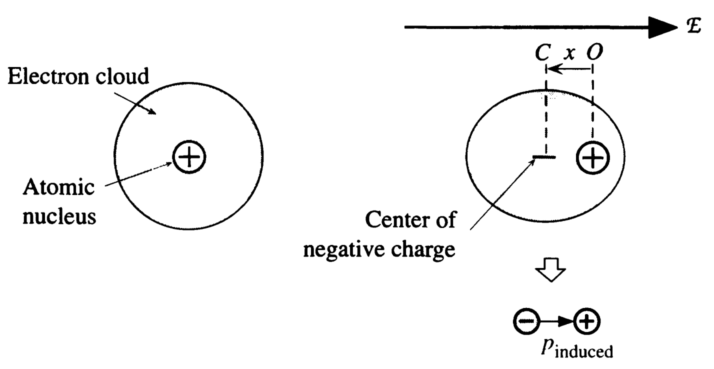


Figure 7.3 The origin of electronic polarization.

(a) A neutral atom in $\mathcal{E} = 0$ (b) Induced dipole moment in a field

from the positive charge center, as shown in Figure 7.3b. This separation of negative and positive charges and the resulting induced dipole moment are termed **polarization.** An atom is said to be **polarized** if it possesses an effective dipole moment, that is, if there is a separation between the centers of negative and positive charge distributions.

The induced dipole moment depends on the electric field causing it. We define a quantity called the **polarizability** α to relate the induced dipole moment p_{induced} to the field $\mathcal{E}$ causing it,

Definition of polarizability

$$p_{\text{induced}} = \alpha\mathcal{E} \qquad [7.4]$$

where α is a coefficient called the polarizability of the atom. It depends on the polarization mechanism. Since the polarization of a neutral atom involves the displacement of electrons, α is called **electronic polarization** and denoted as α_e. Inasmuch as the electrons in an atom are not rigidly fixed, all atoms possess a certain amount of electronic polarizability.

In the absence of an electric field, the center of mass of the orbital motions of the electrons coincides with the positively charged nucleus and the electronic dipole moment is zero. Suppose that the atom has Z number of electrons orbiting the nucleus and all the electrons are contained within a certain spherical region. When an electric field $\mathcal{E}$ is applied, the light electrons become displaced in the opposite direction to $\mathcal{E}$, so their center of mass C is shifted by some distance x with respect to the nucleus O, which we take to be the origin, as shown in Figure 7.3b. As the electrons are "pushed" away by the applied field, the Coulombic attraction between the electrons and nuclear charge "pulls in" the electrons. The force on the electrons, due to $\mathcal{E}$, trying to separate them away from the nuclear charge is $Ze\mathcal{E}$. The restoring force F_r, which is the Coulombic attractive force between the electrons and the nucleus, can be taken to be proportional to the displacement x, provided that the latter is small.[1] The restoring force F_r is obviously zero when C coincides with $O(x = 0)$. We can write

Restoring force

$$F_r = -\beta x$$

---

[1] It may be noticed that even if F_r is a complicated function of x, it can still be expanded in a series in terms of powers of x, that is, x, x^2, x^3, and so on, and for small x only the x term is significant, $F_r = -\beta x$.

where β is a constant and the negative sign indicates that F_r is always directed toward the nucleus O (Figure 7.3b). In equilibrium, the net force on the negative charge is zero or

$$Ze\mathcal{E} = \beta x$$

from which x is known. Therefore the **magnitude** of the induced electronic dipole moment p_e is given by

Electronic polarization

$$p_e = (Ze)x = \left(\frac{Z^2e^2}{\beta}\right)\mathcal{E} \qquad [7.5]$$

As expected, p_e is proportional to the applied field. The electronic dipole moment in Equation 7.5 is valid under static conditions, that is, when the electric field is a dc field. Suppose that we suddenly remove the applied electric field polarizing the atom. There is then only the restoring force $-\beta x$, which always acts to pull the electrons toward the nucleus O. The equation of motion of the negative charge center is then (from force = mass × acceleration)

Equation for simple harmonic motion

$$-\beta x = Zm_e\frac{d^2x}{dt^2}$$

Thus the displacement at any time is

$$x(t) = x_o\cos(\omega_o t)$$

where

Electronic polarization resonance frequency

$$\omega_o = \left(\frac{\beta}{Zm_e}\right)^{1/2} \qquad [7.6]$$

is the oscillation frequency of the center of mass of the electron cloud about the nucleus and x_o is the displacement before the removal of the field. After the removal of the field, the electronic charge cloud executes simple harmonic motion about the nucleus with a natural frequency determined by Equation 7.6; ω_o is called the **electronic polarization resonance frequency.**[2] It is analogous to a mass on a spring being pulled and let go. The system then executes simple harmonic motion. The oscillations of course die out with time. In the atomic case, a sinusoidal displacement implies that the electronic charge cloud has an acceleration

$$\frac{d^2x}{dt^2} = -x_o\omega_o^2\cos(\omega_o t)$$

It is well known from classical electromagnetism that an accelerating charge radiates electromagnetic energy just like a radio antenna. Consequently the oscillating charge

---

[2] The term *natural frequency* refers to a system's characteristic frequency of oscillation when it is excited. A mass attached to a spring and then let go will execute simple harmonic motion with a certain natural frequency ω_o. If we then decide to oscillate this mass with an applied force, the maximum energy transfer will occur when the applied force has the same frequency as ω_o; the system will be put in resonance. ω_o is also a *resonant frequency*. Strictly, $\omega = 2\pi f$ is the angular frequency and f is the frequency. It is quite common to simply refer to ω as a frequency because the literature is dominated by ω; the meaning should be obvious within context.

cloud loses energy, and thus its amplitude of oscillation decreases. (Recall that the average energy is proportional to the square of the amplitude of the displacement.)

From the expression derived for p_e in Equation 7.5, we can find the electronic polarizability α_e from Equation 7.4,

Static electronic polarizability

$$\alpha_e = \frac{Ze^2}{m_e\omega_o^2} \qquad [7.7]$$

EXAMPLE 7.1

ELECTRONIC POLARIZABILITY Consider the electronic polarizability of inert gas atoms. These atoms have closed shells. Their electronic polarizabilities are listed in Table 7.1. For each type of atom calculate the electronic polarization resonant frequency $f_o = \omega_o/2\pi$, and plot α_e and f_o against the number of electrons Z in the atom. What is your conclusion?

SOLUTION

We can use Equation 7.7 to calculate the resonant frequency $f_o = \omega_o/2\pi$. Taking Ar,

$$\omega_o = \left(\frac{Ze^2}{\alpha_e m_e}\right)^{1/2} = \left[\frac{(18)(1.6\times10^{-19})^2}{(1.7\times10^{-40})(9.1\times10^{-31})}\right]^{1/2} = 5.46\times10^{16}\ \text{rad s}^{-1}$$

Table 7.1 Electronic polarizability α_e dependence on Z for the inert element atoms

	Atom					
	He	Ne	Ar	Kr	Xe	Rn*
Z	2	10	18	36	56	
$\alpha_e\times10^{-40}$ (F m^2)	0.18	0.45	1.7	2.7	4.4	5.9
$f_o\times10^{15}$ (Hz)	8.90	12.6	8.69	9.76	9.36	10.2

*Rn (radon) gas is radioactive.

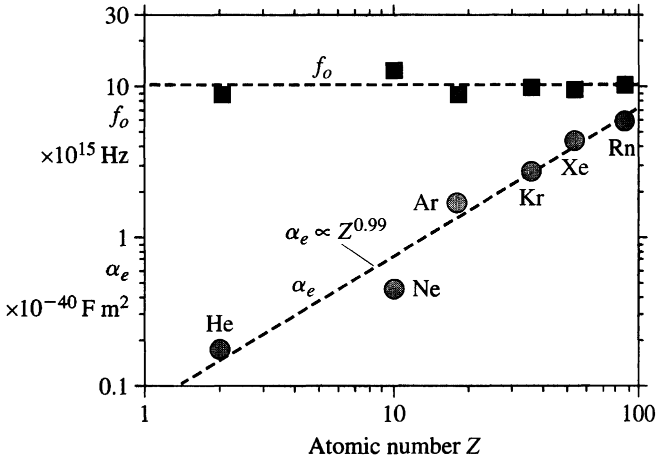


Figure 7.4 Electronic polarizability and its resonance frequency versus the number of electrons in the atom (Z).
The dashed line is the best-fit line.

so that

$$f_o = \frac{\omega_o}{2\pi} = 8.69 \times 10^{15} \text{ Hz}$$

which is listed in Table 7.1, among other f_o calculations for the other atoms. Such frequencies correspond to the field oscillations in UV light, that is, at optical frequencies. For all practical purposes, electronic polarization occurs very rapidly, that is, on a time scale $1/f_o$ or 10^{-15} s, and we can take the static polarizability α_e to remain the same up to optical frequencies.[3]

Figure 7.4 shows the dependence of α_e and f_o on the number of electrons Z. It is apparent that α_e is nearly linearly proportional to Z, whereas f_o is very roughly constant. It is left as an exercise to show that β increases with Z, which is reasonable since the restoring force was defined as the total force between *all* the electrons and the nucleus when the electrons are displaced.

7.1.3 POLARIZATION VECTOR P

When a material is placed in an electric field, the atoms and the molecules of the material become polarized, so we have a distribution of dipole moments in the material. We can visualize this effect with the insertion of the dielectric slab into the parallel plate capacitor, as depicted in Figure 7.5a. The placement of the dielectric slab into an electric field polarizes the molecules in the material. The induced dipole moments all point in the direction of the field. Consider the polarized medium alone, as shown in Figure 7.5b. In the bulk of the material, the dipoles are aligned head to tail. Every positive charge has a negative charge next to it and vice versa. There is therefore no net charge within the bulk. But the positive charges of the dipoles appearing at the right-hand face are not canceled by negative charges of any dipoles at this face. There is therefore a surface charge $+Q_P$ on the right-hand face that results from the polarization of the medium. Similarly, there is a negative charge $-Q_P$ with the same magnitude appearing on the left-hand face due to the negative charges of the dipoles at this face. We see that charges $+Q_P$ and $-Q_P$ appear on the opposite surfaces of a material when it becomes polarized in an electric field, as shown in Figure 7.5c. These charges are **bound** and are a direct result of the polarization of the molecules. They are termed **surface polarization charges.** Figure 7.5c emphasizes this aspect of dielectric behavior in an electric field by showing the dielectric and its polarization charges only.

We represent the polarization of a medium by a quantity called **polarization P,** which is defined as the total dipole moment per unit volume,

Definition of polarization vector

$$\mathbf{P} = \frac{1}{\text{Volume}}[\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_N] \quad [7.8a]$$

where $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N$ are the dipole moments induced at N molecules in the volume. If $\mathbf{p}_{\text{av}}$ is the average dipole moment per molecule, then an equivalent definition of $\mathbf{P}$ is

Definition of polarization vector

$$\mathbf{P} = N\mathbf{p}_{\text{av}} \quad [7.8b]$$

---

[3] Electronic polarization at optical frequencies controls the optical properties such as the refractive index, as will be covered in Chapter 9.

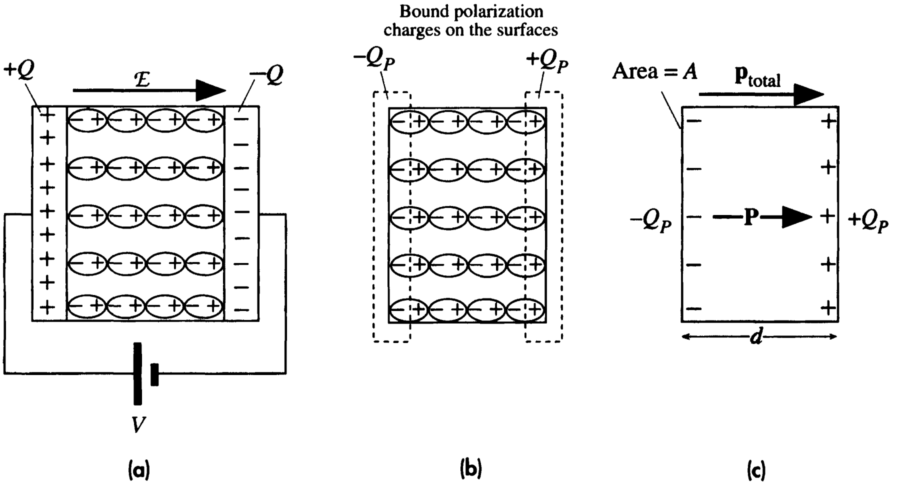


Figure 7.5

(a) When a dielectric is placed in an electric field, bound polarization charges appear on the opposite surfaces.

(b) The origin of these polarization charges is the polarization of the molecules of the medium.

(c) We can represent the whole dielectric in terms of its surface polarization charges $+Q_P$ and $-Q_P$.

where N is the number of molecules per unit volume. There is an important relationship, given below, between **P** and the polarization charges Q_P on the surfaces of the dielectric. It should be emphasized for future discussions that if polarization arises from the effect of the applied field, as shown in Figure 7.5a, which is usually the case, $\mathbf{p}_{\text{av}}$ must be the *average dipole moment per atom in the direction of the applied field.* In that case we often also denote $\mathbf{p}_{\text{av}}$ as the induced average dipole moment per molecule $\mathbf{p}_{\text{induced}}$.

To calculate the polarization **P** for the polarized dielectric in Figure 7.5b, we need to sum all the dipoles in the medium and divide by the volume Ad, as in Equation 7.8a. However, the polarized medium can be simply represented as in Figure 7.5c in terms of surface charge $+Q_P$ and $-Q_P$, which are separated by the thickness distance d. We can view this arrangement as one big dipole moment p_{total} from $-Q_P$ to $+Q_P$. Thus

$$p_{\text{total}} = Q_P d$$

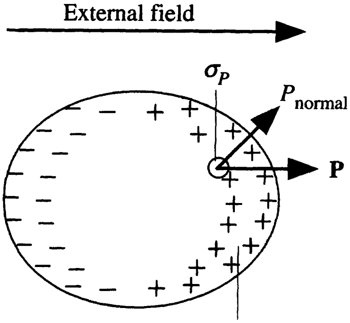
Since the polarization is defined as the total dipole moment per unit volume, the magnitude of **P** is

Polarization and bound surface charge density

$$P = \frac{p_{\text{total}}}{\text{Volume}} = \frac{Q_P d}{Ad} = \frac{Q_P}{A}$$

But Q_P/A is the **surface polarization charge density** σ_P, so

$$P = \sigma_p \qquad [7.9a]$$



Polarization charges on the surface of a polarized medium

Figure 7.6 Polarization charge density on the surface of a polarized medium is related to the normal component of the polarization vector.

Polarization is a vector and Equation 7.9a only gives its magnitude. For the rectangular slab in Figure 7.5c, the direction of P is normal to the surface. For $+\sigma_p$ (right face), it comes out from the surface and for $-\sigma_P$ (left face), it is directed into the surface. Although Equation 7.9a is derived for one specific geometry, the rectangular slab, it can be generalized as follows. *The charge per unit area appearing on the surface of a polarized medium is equal to the component of the polarization vector normal to this surface.* If P_{normal} is the component of **P** normal to the surface where the polarization charge density is σ_P, as shown in Figure 7.6, then,

Polarization and bound surface charge density

$$P_{\text{normal}} = \sigma_p \tag{7.9b}$$

The polarization P induced in a dielectric medium when it is placed in an electric field depends on the field itself. The induced dipole moment per molecule within the medium depends on the electric field by virtue of Equation 7.4. To express the dependence of P on the field $\mathcal{E}$, we define a quantity called the **electric susceptibility** χ_e by

Definition of electric susceptibility

$$P = \chi_e \varepsilon_o \mathcal{E} \tag{7.10}$$

Equation 7.10 shows an *effect* P due to a *cause* $\mathcal{E}$ and the quantity χ_e relates the effect to its cause. Put differently, χ_e acts as a proportionality constant. It may depend on the field itself, in which case the effect is nonlinearly related to the cause. Further, electronic polarizability is defined by

$$p_{\text{induced}} = \alpha_e \mathcal{E}$$

so

$$P = Np_{\text{induced}} = N\alpha_e \mathcal{E}$$

where N is the number of molecules per unit volume. Then from Equation 7.10, χ_e and α_e are related by

Electric susceptibility and polarization

$$\chi_e = \frac{1}{\varepsilon_o} N\alpha_e \tag{7.11}$$

It is important to recognize the difference between *free* and *polarization* (or *bound*) charges. The charges stored on the metal plates in Figure 7.5a are free because they result from the motion of free electrons in the metal. For example both Q_o and Q, before and after the dielectric insertion in Figure 7.1, are free charges that arrive on the plates from the battery. The polarization charges $+Q_P$ and $-Q_P$, on the other hand, are bound to the molecules. They cannot move within the dielectric or on its surface.

The field $\mathcal{E}$ *before* the dielectric was inserted (Figure 7.1a) is given by

$$\mathcal{E} = \frac{V}{d} = \frac{Q_o}{C_o d} = \frac{Q_o}{\varepsilon_o A} = \frac{\sigma_o}{\varepsilon_o} \qquad [7.12]$$

where $\sigma_o = Q_o/A$ is the **free surface charge density** without any dielectric medium between the plates, as in Figure 7.1a.

After the insertion of the dielectric, this field remains the same V/d, but the free charges on the plates are different. The free surface charge on the plates is now Q. In addition there are bound polarization charges on the dielectric surfaces next to the plates, as shown in Figure 7.5a. It is apparent that the flow of current during the insertion of the dielectric, Figure 7.1b, is due to the additional free charges $Q - Q_o$ needed on the capacitor plates to neutralize the opposite polarity polarization charges Q_P appearing on the dielectric surfaces. The total charge (see Figure 7.5a) due to that on the plate plus that appearing on the dielectric surface, $Q - Q_P$, must be the same as before, Q_o, so that the field, as given by Equation 7.12, does not change inside the dielectric, that is,

$$Q - Q_P = Q_o$$

or

$$Q = Q_o + Q_P$$

Dividing by A, defining $\sigma = Q/A$ as the free surface charge density on the plates with the dielectric inserted, and using Equation 7.12, we obtain

$$\sigma = \varepsilon_o \mathcal{E} + \sigma_P$$

Since $\sigma_P = P$ and $P = \chi_e \varepsilon_o \mathcal{E}$, Equations 7.9 and 7.10, we can eliminate σ_P to obtain

$$\sigma = \varepsilon_o (1 + \chi_e) \mathcal{E}$$

From the definition of the relative permittivity in Equation 7.2 we have

$$\varepsilon_r = \frac{Q}{Q_o} = \frac{\sigma}{\sigma_o}$$

so substituting for σ and using Equation 7.12 we obtain

Relative permittivity and electric susceptibility

$$\varepsilon_r = 1 + \chi_e \qquad [7.13]$$

In terms of electronic polarization, from Equation 7.11, this is

Relative permittivity and polarizability

$$\varepsilon_r = 1 + \frac{N \alpha_e}{\varepsilon_o} \qquad [7.14]$$

The significance of Equation 7.14 is that it relates the microscopic polarization mechanism that determines α_e to the macroscopic property ε_r.

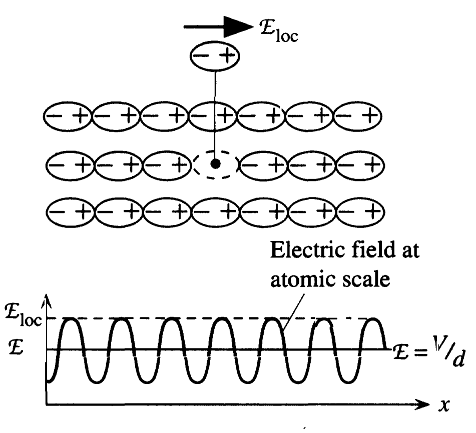


Figure 7.7 The electric field inside a polarized dielectric at the atomic scale is not uniform.
The local field is the actual field that acts on a molecule. It can be calculated by removing that molecule and evaluating the field at that point from the charges on the plates and the dipoles surrounding the point.

7.1.4 LOCAL FIELD $\mathcal{E}_{loc}$ AND CLAUSIUS–MOSSOTTI EQUATION

Equation 7.14, which relates ε_r to electronic polarizability α_e is only approximate because it assumes that the field acting on an individual atom or molecule is the field $\mathcal{E}$, which is assumed to be uniform within the dielectric. In other words, the induced polarization, $p_{\text{induced}} \propto \mathcal{E}$. However, the induced polarization depends on the actual field experienced by the molecule. It is apparent from Figure 7.5a that there are polarized molecules within the dielectric with their negative and positive charges separated so that the field is not constant *on the atomic scale* as we move through the dielectric. This is depicted in Figure 7.7. The field experienced by an individual molecule is actually different than $\mathcal{E}$, which represents the average field in the dielectric. As soon as the dielectric becomes polarized, the field at some arbitrary point depends not only on the charges on the plates (Q) but also on the orientations of all the other dipoles around this point in the dielectric. When averaged over some distance, say a few thousand molecules, this field becomes $\mathcal{E}$, as shown in Figure 7.7.

The actual field experienced by a molecule in a dielectric is defined as the **local field** and denoted by $\mathcal{E}_{loc}$. It depends not only on the free charges on the plates but also on the arrangement of all the polarized molecules around this point. In evaluating $\mathcal{E}_{loc}$ we simply remove the molecule from this point and calculate the field at this point coming from all sources, including neighboring polarized molecules, as visualized in Figure 7.7. $\mathcal{E}_{loc}$ will depend on the amount of polarization the material has experienced. The greater the polarization, the greater is the local field because there are bigger dipoles around this point. $\mathcal{E}_{loc}$ depends on the arrangement of polarized molecules around the point of interest and hence depends on the crystal structure. In the simplest case of a material with a cubic crystal structure, or a liquid (no crystal structure), the local field $\mathcal{E}_{loc}$ acting on a molecule increases with polarization as[4]

$$\mathcal{E}_{loc} = \mathcal{E} + \frac{1}{3\varepsilon_o}P \qquad [7.15]$$

Lorentz local field in dielectrics

---

[4] This field is called the **Lorentz field** and the proof, though not difficult, is not necessary for the present introductory treatment of dielectrics. This local field expression does not apply to dipolar dielectrics discussed in Section 7.3.2.

Equation 7.15 is called the **Lorentz field.** The induced polarization in the molecule now depends on this local field $\mathcal{E}_{\text{loc}}$ rather than the average field $\mathcal{E}$. Thus

$$p_{\text{induced}} = \alpha_e \mathcal{E}_{\text{loc}}$$

The fundamental definition of electric susceptibility by the equation

$$P = \chi_e \varepsilon_o \mathcal{E}$$

is unchanged, which means that $\varepsilon_r = 1 + \chi_e$, Equation 7.13, remains intact. The polarization is defined by $P = Np_{\text{induced}}$, and p_{induced} can be related to $\mathcal{E}_{\text{loc}}$ and hence to $\mathcal{E}$ and P. Then

$$P = (\varepsilon_r - 1)\varepsilon_o \mathcal{E}$$

can be used to eliminate $\mathcal{E}$ and P and obtain a relationship between ε_r and α_e. This is the **Clausius–Mossotti equation,**

Clausius–Mossotti equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_o} \qquad [7.16]$$

This equation allows the calculation of the macroscopic property ε_r from microscopic polarization phenomena, namely, α_e.

EXAMPLE 7.2

ELECTRONIC POLARIZABILITY OF A VAN DER WAALS SOLID The electronic polarizability of the Ar atom is 1.7×10^{-40} F m^2. What is the static dielectric constant of solid Ar (below 84 K) if its density is 1.8 g cm^{-3}?

SOLUTION

To calculate ε_r we need the number of Ar atoms per unit volume N from the density d. If $M_{\text{at}} = 39.95$ is the relative atomic mass of Ar and N_A is Avogadro's number, then

$$N = \frac{N_A d}{M_{\text{at}}} = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(1.8\ \text{g cm}^{-3})}{(39.95\ \text{g mol}^{-1})} = 2.71 \times 10^{22}\ \text{cm}^{-3}$$

with $N = 2.71 \times 10^{28}$ m^{-3} and $\alpha_e = 1.7 \times 10^{-40}$ F m^2, we have

$$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_o} = 1 + \frac{(2.71 \times 10^{28})(1.7 \times 10^{-40})}{(8.85 \times 10^{-12})} = 1.52$$

If we use the Clausius–Mossotti equation, we get

$$\varepsilon_r = \frac{1 + \dfrac{2N\alpha_e}{3\varepsilon_o}}{1 - \dfrac{N\alpha_e}{3\varepsilon_o}} = 1.63$$

The two values are different by about 7 percent. The simple relationship in Equation 7.14 underestimates the relative permittivity.

7.2 ELECTRONIC POLARIZATION: COVALENT SOLIDS

When a field is applied to a solid substance, the constituent atoms or molecules become polarized, as we visualized in Figure 7.5a. The electron clouds within each atom become shifted by the field, and this gives rise to **electronic polarization.** This type of electronic polarization within an atom, however, is quite small compared with the polarization due to the valence electrons in the covalent bonds within the solid. For example, in crystalline silicon, there are electrons shared with neighboring Si atoms in covalent bonds, as shown in Figure 7.8a. These valence electrons form bonds (*i.e.*, become shared) between the Si atoms because they are already loosely bound to their parent atoms. If this were not the case, the solid would be a van der Waals solid with atoms held together by secondary bonds (*e.g.*, solid Ar below 83.8 K). In the covalent solid, the valence electrons therefore are not rigidly tied to the ionic cores left in the Si atoms. Although intuitively we often view these valence electrons as living in covalent bonds between the ionic Si cores, they nonetheless belong to the whole crystal because they can tunnel from bond to bond and exchange places with each other. We refer to their wavefunctions as delocalized, that is, not localized to any particular Si atom. When an electric field is applied, the negative charge distribution associated with these valence electrons becomes readily shifted with respect to the positive charges of the ionic Si cores, as depicted in Figure 7.8b and the crystal exhibits polarization, or develops a polarization vector. One can appreciate the greater flexibility of electrons in covalent bonds compared with those in individual ionic cores by comparing the energy involved in freeing each. It takes perhaps 1–2 eV to break a covalent bond to free the valence electron, but it takes more than 10 eV to free an electron from an individual ionic Si core. Thus, the valence electrons in the bonds readily respond to an applied field and become displaced. This type of electronic polarization, due to the displacement of electrons in covalent bonds, is responsible for the large dielectric constants of covalent crystals. For example $\varepsilon_r = 11.9$ for the Si crystal and $\varepsilon_r = 16$ for the Ge crystal.

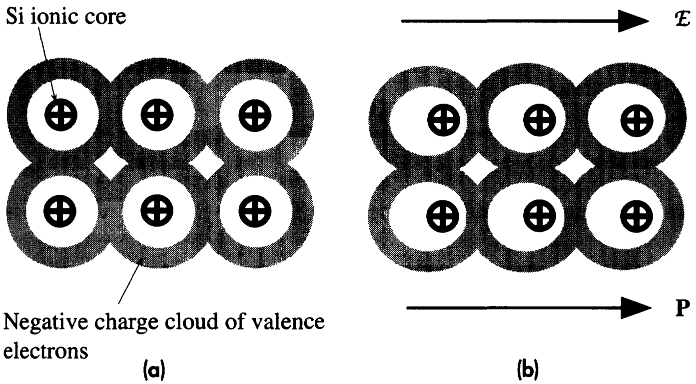


Figure 7.8

(a) Valence electrons in covalent bonds in the absence of an applied field.

(b) When an electric field is applied to a covalent solid, the valence electrons in the covalent bonds are shifted very easily with respect to the positive ionic cores. The whole solid becomes polarized due to the collective shift in the negative charge distribution of the valence electrons.

EXAMPLE 7.3 **ELECTRONIC POLARIZABILITY OF COVALENT SOLIDS** Consider a pure Si crystal that has $\varepsilon_r = 11.9$.

a. What is the electronic polarizability due to valence electrons per Si atom (if one could portion the observed crystal polarization to individual atoms)?

b. Suppose that a Si crystal sample is electroded on opposite faces and has a voltage applied across it. By how much is the local field greater than the applied field?

c. What is the resonant frequency f_o corresponding to ω_o?

From the density of the Si crystal, the number of Si atoms per unit volume, N, is given as $5 \times 10^{28}\,\text{m}^{-3}$.

SOLUTION

a. Given the number of Si atoms, we can apply the Clausius–Mossotti equation to find α_e

$$\alpha_e = \frac{3\varepsilon_o}{N}\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{3(8.85 \times 10^{-12})}{(5 \times 10^{28})}\frac{11.9 - 1}{11.9 + 2} = 4.17 \times 10^{-40}\,\text{F m}^2$$

This is larger, for example, than the electronic polarizability of an isolated Ar atom, which has more electrons. If we were to take the inner electrons in each Si atom as very roughly representing Ne, we would expect their contribution to the overall electronic polarizability to be roughly the same as the Ne atom, which is 0.45×10^{-40} F m^2.

b. The local field is

$$\mathcal{E}_{\text{loc}} = \mathcal{E} + \frac{1}{3\varepsilon_o}P$$

But, by definition,

$$P = \chi_e\varepsilon_o\mathcal{E} = (\varepsilon_r - 1)\varepsilon_o\mathcal{E}$$

Substituting for P,

$$\mathcal{E}_{\text{loc}} = \mathcal{E} + \frac{1}{3}(\varepsilon_r - 1)\mathcal{E}$$

so the local field with respect to the applied field is

$$\frac{\mathcal{E}_{\text{loc}}}{\mathcal{E}} = \frac{1}{3}(\varepsilon_r + 2) = 4.63$$

The local field is a factor of 4.63 greater than the applied field.

c. Since polarization is due to valence electrons and there are four per Si atom, we can use Equation 7.7,

$$\omega_o = \left(\frac{Ze^2}{m_e\alpha_e}\right)^{1/2} = \left[\frac{4(1.6 \times 10^{-19})^2}{(9.1 \times 10^{-31})(4.17 \times 10^{-40})}\right]^{1/2} = 1.65 \times 10^{16}\,\text{rad s}^{-1}$$

The corresponding resonant frequency is $\omega_o/2\pi$ or 2.6×10^{15} Hz, which is typically associated with electromagnetic waves of wavelength in the ultraviolet region.

7.3 POLARIZATION MECHANISMS

In addition to electronic polarization, we can identify a number of other polarization mechanisms that may also contribute to the relative permittivity.

7.3.1 IONIC POLARIZATION

This type of polarization occurs in ionic crystals such as NaCl, KCl, and LiBr. The ionic crystal has distinctly identifiable ions, for example, Na^+ and Cl^-, located at well-defined lattice sites, so each pair of oppositely charged neighboring ions has a dipole moment. As an example, we consider the one-dimensional NaCl crystal depicted as a chain of alternating Na^+ and Cl^- ions in Figure 7.9a. In the absence of an applied field, the solid has no net polarization because the dipole moments of equal magnitude are lined up head to head and tail to tail so that the net dipole moment is zero. The dipole moment p_+ in the positive x direction has the same magnitude as p_- in the negative x direction, so the net dipole moment

$$p_{\text{net}} = p_+ - p_- = 0$$

In the presence of a field $\mathcal{E}$ along the x direction, however, the Cl^- ions are pushed in the $-x$ direction and the Na^+ ions in the $+x$ direction about their equilibrium positions. Consequently, the dipole moment p_+ in the $+x$ direction *increases* to p'_+ and the dipole moment p_- *decreases* to p'_-, as shown in Figure 7.9b. The net dipole moment is now no longer zero. The net dipole moment, or the average dipole moment, per ion pair is now $(p'_+ - p'_-)$, which depends on the electric field $\mathcal{E}$. Thus the induced average dipole moment per ion pair p_{av} depends on the field $\mathcal{E}$. The ionic polarizability α_i is defined in terms of the local field experienced by the ions,

$$p_{\text{av}} = \alpha_i \mathcal{E}_{\text{loc}} \qquad [7.17]$$

Ionic polarizability

The larger the α_i, the greater the induced dipole moment. Generally, α_i is larger than the electronic polarizability α_e by a factor of 10 or more, which leads to ionic solids having large dielectric constants. The polarization P exhibited by the ionic solid

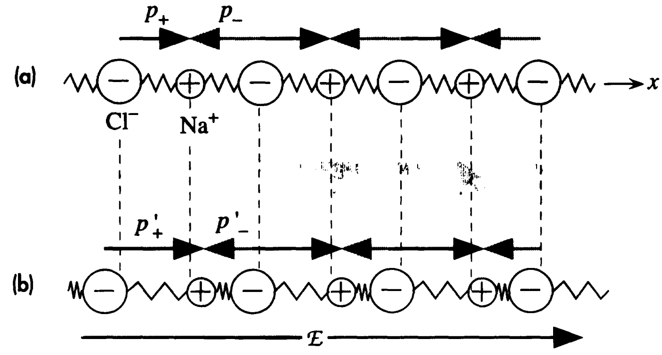


Figure 7.9

(a) A NaCl chain in the NaCl crystal without an applied field. Average or net dipole moment per ion is zero.

(b) In the presence of an applied field, the ions become slightly displaced, which leads to a net average dipole moment per ion.

is therefore given by

$$P = N_i p_{\text{av}} = N_i \alpha_i \mathcal{E}_{\text{loc}}$$

where N_i is the number of ion pairs per unit volume. By relating the local field to $\mathcal{E}$ and using

$$P = (\varepsilon_r - 1)\varepsilon_o \mathcal{E}$$

Clausius–Mossotti equation for ionic polarization

we can again obtain the Clausius–Mossotti equation, but now due to ionic polarization,

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_o} N_i \alpha_i \qquad [7.18]$$

Each ion also has a core of electrons that become displaced in the presence of an applied field with respect to their positive nuclei and therefore also contribute to the polarization of the solid. This electronic polarization simply adds to the ionic polarization. Its magnitude is invariably much smaller than the ionic contribution in these solids.

7.3.2 ORIENTATIONAL (DIPOLAR) POLARIZATION

Certain molecules possess permanent dipole moments. For example, the HCl molecule shown in Figure 7.10a has a permanent dipole moment p_o from the Cl^- ion to the H^+ ion. In the liquid or gas phases, these molecules, in the absence of an electric field, are randomly oriented as a result of thermal agitation, as shown in Figure 7.10b. When an electric field $\mathcal{E}$ is applied, $\mathcal{E}$ tries to align the dipoles parallel to itself, as depicted in Figure 7.10c. The Cl^- and H^+ charges experience forces in opposite directions. But the nearly rigid bond between Cl^- and H^+ holds them together, which means that the

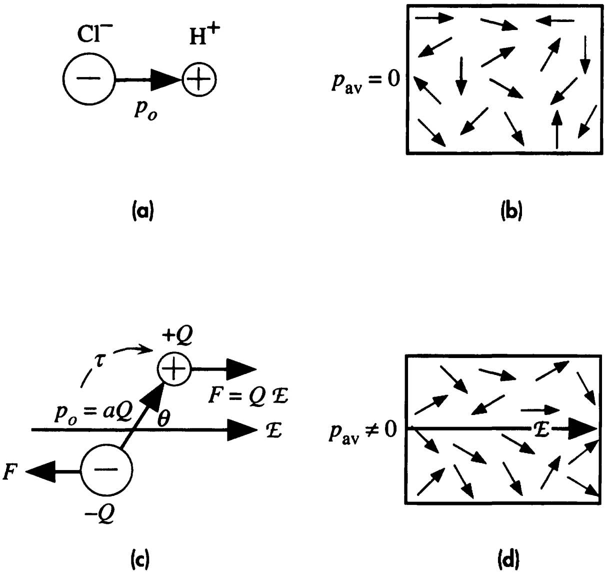


Figure 7.10

(a) A HCl molecule possesses a permanent dipole moment p_o.

(b) In the absence of a field, thermal agitation of the molecules results in zero net average dipole moment per molecule.

(c) A dipole such as HCl placed in a field experiences a torque that tries to rotate it to align p_o with the field $\mathcal{E}$.

(d) In the presence of an applied field, the dipoles try to rotate to align with the field against thermal agitation. There is now a net average dipole moment per molecule along the field.

molecule experiences a torque τ about its center of mass.[5] This torque acts to rotate the molecule to align p_o with $\mathcal{E}$. If all the molecules were to simply rotate and align with the field, the polarization of the solid would be

$$P = Np_o$$

where N is the number of molecules per unit volume. However, due to their thermal energy, the molecules move around randomly and collide with each other and with the walls of the container. These collisions destroy the dipole alignments. Thus the thermal energy tries to randomize the orientations of the dipole moments. A snapshot of the dipoles in the material in the presence of a field can be pictured as in Figure 7.10d in which the dipoles have different orientations. There is, nonetheless, a net average dipole moment per molecule p_{av} that is finite and directed along the field. Thus the material exhibits net polarization, which leads to a dielectric constant that is determined by this **orientational polarization.**

To find the induced average dipole moment p_{av} along $\mathcal{E}$, we need to know the average potential energy E_{dip} of a dipole placed in a field $\mathcal{E}$ and how this compares with the average thermal energy $\frac{5}{2}kT$ per molecule as in the present case of five degrees of freedom. E_{dip} represents the average external work done by the field in aligning the dipoles with the field. If $\frac{5}{2}kT$ is much greater than E_{dip}, then the average thermal energy of collisions will prevent any dipole alignment with the field. If, however, E_{dip} is much greater than $\frac{5}{2}kT$, then the thermal energy is insufficient to destroy the dipole alignments.

A dipole at an angle θ to the field experiences a torque τ that tries to rotate it, as shown in Figure 7.10c. Work done dW by the field in rotating the dipole by $d\theta$ is $\tau\, d\theta$ (as in $F\,dx$). This work dW represents a small change dE in the potential energy of the dipole. No work is done if the dipole is already aligned with $\mathcal{E}$, when $\theta = 0$, which corresponds to the minimum in PE. On the other hand, maximum work is done when the torque has to rotate the dipole from $\theta = 180°$ to $\theta = 0°$ (either clockwise or counterclockwise, it doesn't matter). The torque experienced by the dipole, according to Figure 7.10c, is given by

Torque on a dipole

$$\tau = (F \sin\theta)a \qquad \text{or} \qquad \mathcal{E}p_o \sin\theta$$

where

$$p_o = aQ$$

If we take $PE = 0$ when $\theta = 0$, then the maximum PE is when $\theta = 180°$, or

$$E_{\text{max}} = \int_0^{\pi} p_o \mathcal{E} \sin\theta\, d\theta = 2p_o\mathcal{E}$$

The average dipole potential energy is then $\frac{1}{2}E_{\text{max}}$ or $p_o\mathcal{E}$. For orientational polarization to be effective, this energy must be greater than the average thermal energy. The average dipole moment p_{av} along $\mathcal{E}$ is directly proportional to the magnitude of p_o itself and also proportional to the average dipole energy to average thermal energy

---

[5] The oppositely directed forces also slightly stretch the $\text{Cl}^-\text{–H}^+$ bond, but we neglect this effect.

ratio, that is,

$$p_{\text{av}} \propto p_o \frac{p_o \mathcal{E}}{\frac{5}{2}kT}$$

If we were to do the calculation properly using Boltzmann statistics for the distribution of dipole energies among the molecules, that is, the probability that the dipole has an energy E is proportional to $\exp(-E/kT)$, then we would find that when $p_o E < kT$ (generally the case),

Average dipole moment in orientational polarization

$$p_{\text{av}} = \frac{1}{3}\frac{p_o^2 \mathcal{E}}{kT} \tag{7.19}$$

It turns out that the intuitively derived expression for p_{av} is roughly the same as Equation 7.19. Strictly, of course, we should use the local field acting on each molecule, in which case $\mathcal{E}$ is simply replaced by $\mathcal{E}_{\text{loc}}$. From Equation 7.19 we can define a **dipolar orientational polarizability** α_d per molecule by

Dipolar orientational polarizability

$$\alpha_d = \frac{1}{3}\frac{p_o^2}{kT} \tag{7.20}$$

It is apparent that, in contrast to the electronic and ionic polarization, dipolar orientational polarization is strongly temperature dependent. α_d decreases with temperature, which means that the relative permittivity ε_r also decreases with temperature. Dipolar orientational polarization is normally exhibited by polar liquids (*e.g.*, water, alcohol, acetone, and various electrolytes) and polar gases (*e.g.*, gaseous HCl and steam). It can also occur in solids if there are permanent dipoles within the solid structure, even if dipolar rotation involves a discrete jump of an ion from one site to another, such as in various glasses.

7.3.3 INTERFACIAL POLARIZATION

Interfacial polarization occurs whenever there is an accumulation of charge at an interface between two materials or between two regions within a material. The simplest example is interfacial polarization due to the accumulation of charges in the dielectric near one of the electrodes, as depicted in Figure 7.11a and b. Invariably materials, however perfect, contain crystal defects, impurities, and various mobile charge carriers such as electrons (*e.g.*, from donor-type impurities), holes, or ionized host or impurity ions. In the particular example in Figure 7.11a, the material has an equal number of positive ions and negative ions, but the positive ions are assumed to be far more mobile. For example, if present, the H^+ ion (which is a proton) and the Li^+ ion in ceramics and glasses are more mobile than negative ions in the structure because they are relatively small. Under the presence of an applied field, these positive ions migrate to the negative electrode. The positive ions, however, cannot leave the dielectric and enter the crystal structure of the metal electrode. They therefore simply pile up at the interface and give rise to a positive space charge near the electrode. These positive charges at the interface attract more electrons to the negative electrode. This additional charge on the electrode,

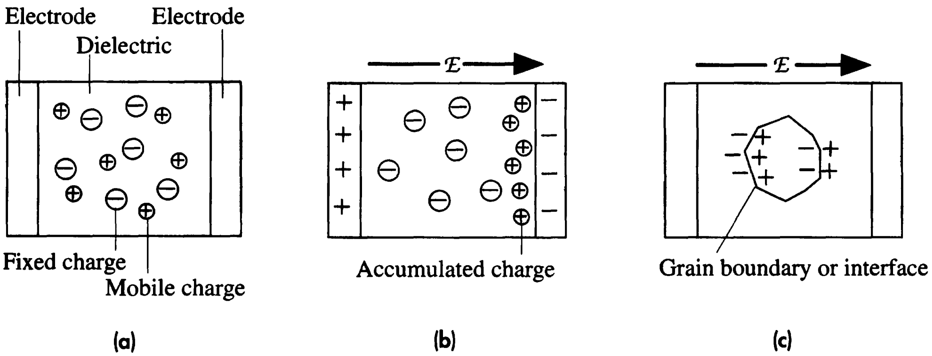


Figure 7.11

(a) A crystal with equal number of mobile positive ions and fixed negative ions. In the absence of a field, there is no net separation between all the positive charges and all the negative charges.

(b) In the presence of an applied field, the mobile positive ions migrate toward the negative electrode and accumulate there. There is now an overall separation between the negative charges and positive charges in the dielectric. The dielectric therefore exhibits interfacial polarization.

(c) Grain boundaries and interfaces between different materials frequently give rise to interfacial polarization.

of course, appears as an increase in the dielectric constant. The term **interfacial polarization** arises because the positive charges accumulating at the interface and the remainder of negative charges in the bulk together constitute dipole moments that appear in the polarization vector **P** (**P** sums all the dipoles within the material per unit volume).

Another typical interfacial polarization mechanism is the trapping of electrons or holes at defects at the crystal surface, at the interface between the crystal and the electrode. In this case we can view the positive charges in Figure 7.11a as holes and negative charges as immobile ionized acceptors. We assume that the contacts are blocking and do not allow electrons or holes to be injected, that is, exchanged between the electrodes and the dielectric. In the presence of a field, the holes drift to the negative electrode and become trapped in defects at the interface, as in Figure 7.11b.

Grain boundaries frequently lead to interfacial polarization as they can trap charges migrating under the influence of an applied field, as indicated in Figure 7.11c. Dipoles between the trapped charges increase the polarization vector. Interfaces also arise in heterogeneous dielectric materials, for example, when there is a dispersed phase within a continuous phase. The principle is then the same as schematically illustrated in Figure 7.11c.

7.3.4 TOTAL POLARIZATION

In the presence of electronic, ionic, and dipolar polarization mechanisms, the average induced dipole moment per molecule will be the sum of all the contributions in terms of the local field,

Total induced dipole moment

$$p_{\mathrm{av}} = \alpha_e \mathcal{E}_{\mathrm{loc}} + \alpha_i \mathcal{E}_{\mathrm{loc}} + \alpha_d \mathcal{E}_{\mathrm{loc}}$$

Table 7.2 Typical examples of polarization mechanisms

Example	Polarization	Static ε_r	Comment
Ar gas	Electronic	1.0005	Small N in gases: $\varepsilon_r \approx 1$
Ar liquid ($T < 87.3$ K)	Electronic	1.53	van der Waals bonding
Si crystal	Electronic polarization due to valence electrons	11.9	Covalent solid; bond polarization
NaCl crystal	Ionic	5.90	Ionic crystalline solid
CsCl crystal	Ionic	7.20	Ionic crystalline solid
Water	Orientational	80	Dipolar liquid
Nitromethane (27 °C)	Orientational	34	Dipolar liquid
PVC (polyvinyl chloride)	Orientational	7	Dipole orientations partly hindered in the solid

Each effect adds linearly to the net dipole moment per molecule, a fact verified by experiments. Interfacial polarization cannot be simply added to the above equation as $\alpha_{if}\mathcal{E}_{\text{loc}}$ because it occurs at interfaces and cannot be put into an average polarization per molecule in the bulk. Further, the fields are not well defined at the interfaces. In addition, we *cannot* use the simple Lorentz local field approximation for dipolar materials. That is, the Clausius–Mossotti equation does not work with dipolar dielectrics and the calculation of the local field is quite complicated. The dielectric constant ε_r under **electronic** and **ionic polarizations,** however, can be obtained from

Clausius–Mossotti equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_o}(N_e\alpha_e + N_i\alpha_i) \tag{7.21}$$

Table 7.2 summarizes the various polarization mechanisms and the corresponding static (or very low frequency) dielectric constant. Typical examples where one mechanism dominates over others are also listed.

EXAMPLE 7.4 **IONIC AND ELECTRONIC POLARIZABILITY** Consider the CsCl crystal which has one Cs^+–Cl^- pair per unit cell and a lattice parameter a of 0.412 nm. The electronic polarizability of Cs^+ and Cl^- ions is 3.35×10^{-40} F m^2 and 3.40×10^{-40} F m^2, respectively, and the mean ionic polarizability per ion pair is 6×10^{-40} F m^2. What is the dielectric constant at low frequencies and that at optical frequencies?

SOLUTION

The CsCl structure has one cation (Cs^+) and one anion (Cl^-) in the unit cell. Given the lattice parameter $a = 0.412 \times 10^{-9}$ m, the number of ion pairs N_i per unit volume is $1/a^3 = 1/(0.412 \times 10^{-9}\ \text{m})^3 = 1.43 \times 10^{28}\ \text{m}^{-3}$. N_i is also the concentration of cations and anions individually. From the Clausius–Mossotti equation,

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_o}[N_i\alpha_e(\text{Cs}^+) + N_i\alpha_e(\text{Cl}^-) + N_i\alpha_i]$$

That is,

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{(1.43 \times 10^{28}\ \text{m}^{-3})(3.35 \times 10^{-40} + 3.40 \times 10^{-40} + 6 \times 10^{-40}\ \text{F m}^2)}{3(8.85 \times 10^{-12}\ \text{F m}^{-1})}$$

Solving for ε_r, we find $\varepsilon_r = 7.56$.

At high frequencies—that is, near-optical frequencies—the ionic polarization is too sluggish to allow ionic polarization to contribute to ε_r. Thus, ε_{rop}, relative permittivity at optical frequencies, is given by

$$\frac{\varepsilon_{rop} - 1}{\varepsilon_{rop} + 2} = \frac{1}{3\varepsilon_o}[N_i\alpha_e(\text{Cs}^+) + N_i\alpha_e(\text{Cl}^-)]$$

That is,

$$\frac{\varepsilon_{rop} - 1}{\varepsilon_{rop} + 2} = \frac{(1.43 \times 10^{28}\ \text{m}^{-3})(3.35 \times 10^{-40} + 3.40 \times 10^{-40}\ \text{F m}^2)}{3(8.85 \times 10^{-12}\ \text{F m}^{-1})}$$

Solving for ε_{rop}, we find $\varepsilon_{rop} = 2.71$. Note that experimental values are $\varepsilon_r = 7.20$ at low frequencies and $\varepsilon_{rop} = 2.62$ at high frequencies, very close to calculated values.

7.4 FREQUENCY DEPENDENCE: DIELECTRIC CONSTANT AND DIELECTRIC LOSS

7.4.1 DIELECTRIC LOSS

The static dielectric constant is an effect of polarization under dc conditions. When the applied field, or the voltage across a parallel plate capacitor, is a sinusoidal signal, then the polarization of the medium under these ac conditions leads to an ac dielectric constant that is generally different than the static case. As an example we will consider orientational polarization involving dipolar molecules. The sinusoidally varying field changes magnitude and direction continuously, and it tries to line up the dipoles one way and then the other way and so on. If the instantaneous induced dipole moment p per molecule can instantaneously follow the field variations, then at any instant

$$p = \alpha_d \mathcal{E} \tag{7.22}$$

and the polarizability α_d has its expected maximum value from dc conditions, that is,

$$\alpha_d = \frac{p_o^2}{3kT} \tag{7.23}$$

There are two factors opposing the immediate alignment of the dipoles with the field. First is that thermal agitation tries to randomize the dipole orientations. Collisions in the gas phase, random jolting from lattice vibrations in the liquid and solid phases, for example, aid the randomization of the dipole orientations. Second, the molecules rotate in a viscous medium by virtue of their interactions with neighbors, which is particularly strong in the liquid and solid states and means that the dipoles cannot respond instantaneously to the changes in the applied field. If the field changes too

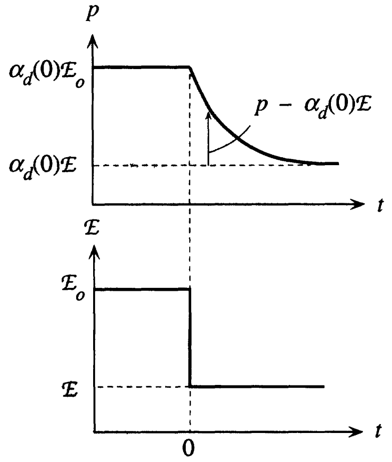


Figure 7.12 The applied dc field is suddenly changed from $\mathcal{E}_o$ to $\mathcal{E}$ at time $t = 0$.
The induced dipole moment p has to decrease from $\alpha_d(0)\mathcal{E}_o$ to a final value of $\alpha_d(0)\mathcal{E}$. The decrease is achieved by random collisions of molecules in the gas.

rapidly, then the dipoles cannot follow the field and, as a consequence, remain randomly oriented. At high frequencies, therefore, α_d will be zero as the field cannot induce a dipole moment. At low frequencies, of course, the dipoles can respond rapidly to follow the field and α_d has its maximum value. It is clear that α_d changes from its maximum value in Equation 7.23 to zero as the frequency of the field is increased. We need to find the behavior of α_d as a function of frequency ω so that we can determine the dielectric constant ε_r by the Clausius–Mossotti equation.

Suppose that after a prolonged application, corresponding to dc conditions, the applied field across the dipolar gaseous medium is suddenly decreased from $\mathcal{E}_o$ to $\mathcal{E}$ at a time we define as zero, as shown in Figure 7.12. The field $\mathcal{E}$ is smaller than $\mathcal{E}_o$, so the induced dc dipole moment per molecule should be smaller and given by $\alpha_d(0)\mathcal{E}$ where $\alpha_d(0)$ is α_d at $\omega = 0$, dc conditions. Therefore, the induced dipole moment per molecule has to decrease, or *relax,* from $\alpha_d(0)\mathcal{E}_o$ to $\alpha_d(0)\mathcal{E}$. In a gas medium the molecules would be moving around randomly and their collisions with each other and the walls of the container randomize the induced dipole per molecule. Thus the decrease, or the **relaxation process,** in the induced dipole moment is achieved by random collisions. Assuming that τ is the average time, called the **relaxation time,** between molecular collisions, then this is the mean time it takes per molecule to randomize the induced dipole moment. If p is the instantaneous induced dipole moment, then $p - \alpha_d(0)\mathcal{E}$ is the *excess* dipole moment, which must eventually disappear to zero through random collisions as $t \to \infty$. It would take an average τ seconds to eliminate the excess dipole moment $p - \alpha_d(0)\mathcal{E}$. The rate at which the induced dipole moment is changing is then $-[p - \alpha_d(0)\mathcal{E}]/\tau$, where the negative sign represents a decrease. Thus,

Dipolar relaxation equation

$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)\mathcal{E}}{\tau} \qquad [7.24]$$

Although we did not derive Equation 7.24 rigorously, it is nonetheless a good first-order description of the behavior of the induced dipole moment per molecule in a dipolar medium. Equation 7.24 can be used to obtain the dipolar polarizability under ac conditions. For an ac field, we would write

$$\mathcal{E} = \mathcal{E}_o \sin(\omega t)$$

and solve Equation 7.24, but in engineering we prefer to use an exponential representation for the field

Applied field

$$\mathcal{E} = \mathcal{E}_o \exp(j\omega t)$$

as in ac voltages. In this case the impedance of a capacitor C and an inductor L become $1/j\omega C$ and $j\omega L$, where j represents a phase shift of 90°. With $\mathcal{E} = \mathcal{E}_o \exp(j\omega t)$ in Equation 7.24, we have

Dipole relaxation equation

$$\frac{dp}{dt} = -\frac{p}{\tau} + \frac{\alpha_d(0)}{\tau}\mathcal{E}_o \exp(j\omega t) \qquad [7.25]$$

Solving this we find the induced dipole moment as

$$p = \alpha_d(\omega)\mathcal{E}_o \exp(j\omega t)$$

where $\alpha_d(\omega)$ is given by

Orientational polarizability and frequency

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau} \qquad [7.26]$$

and represents the orientational polarizability under ac field conditions. Polarizability $\alpha_d(\omega)$ is a complex number that indicates that p and $\mathcal{E}$ are out of phase.[6] Put differently, if N is the number of molecules per unit volume, $P = Np$ and $\mathcal{E}$ are out of phase, as indicated in Figure 7.13a. At low frequencies, $\omega\tau \ll 1$, $\alpha_d(\omega)$ is nearly $\alpha_d(0)$, and p is in phase with $\mathcal{E}$. The rate of relaxation $1/\tau$ is much faster than the frequency of the field or the rate at which the polarization is being changed; p then closely follows $\mathcal{E}$. At very high frequencies, $\omega\tau \gg 1$, the rate of relaxation $1/\tau$ is much slower than the frequency of the field and p can no longer follow the variations in the field.

We can easily obtain the dielectric constant ε_r from $\alpha_d(\omega)$ by using Equation 7.14, which then leads to a complex number for ε_r since α_d itself is a complex number. By convention, we generally write the **complex dielectric constant** as

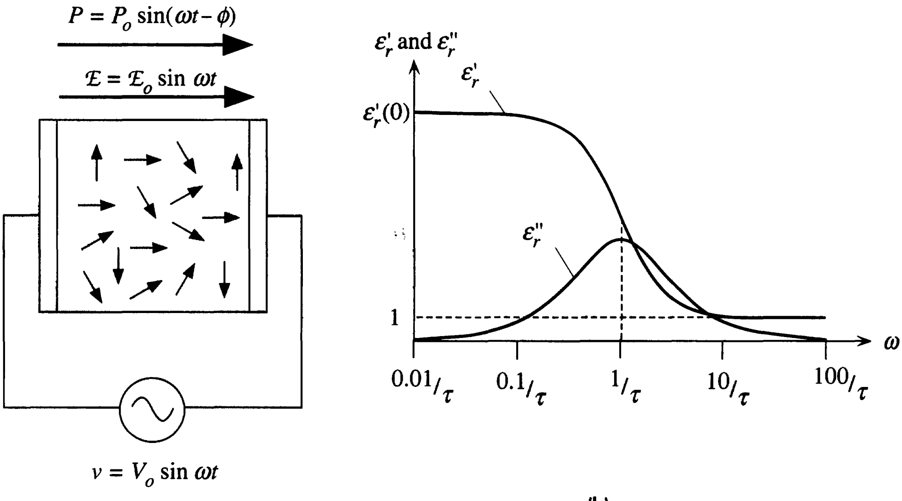
Complex relative permittivity

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \qquad [7.27]$$

where ε_r' is the real part and ε_r'' is the imaginary part, both being frequency dependent, as shown in Figure 7.13b. The real part ε_r' decreases from its maximum value $\varepsilon_r'(0)$, corresponding to $\alpha_d(0)$, to 1 at high frequencies when $\alpha_d = 0$ as $\omega \to \infty$ in Equation 7.26. The imaginary part $\varepsilon_r''(\omega)$ is zero at low and high frequencies but peaks when $\omega\tau = 1$ or when $\omega = 1/\tau$. The real part ε_r' represents the relative permittivity that we would use in calculating the capacitance, as for example in $C = \varepsilon_r\varepsilon_o A/d$. The imaginary part $\varepsilon_r''(\omega)$ represents the energy lost in the dielectric medium as the dipoles are oriented against random collisions one way and then the other way and so on by the field. Consider

---

[6] The polarization P lags behind $\mathcal{E}$ by some angle ϕ, that is determined by Equation 7.26 as shown in Figure 7.13.



(a)

(b)

Figure 7.13

(a) An ac field is applied to a dipolar medium. The polarization P ($P = Np$) is out of phase with the ac field.

(b) The relative permittivity is a complex number with real (ε_r') and imaginary (ε_r'') parts that exhibit relaxation at $\omega \approx 1/\tau$.

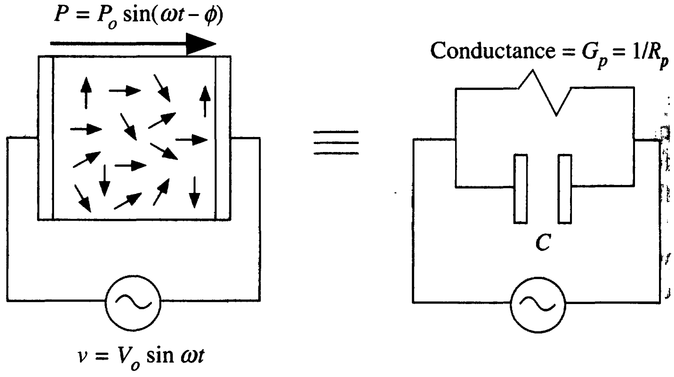


Figure 7.14 The dielectric medium behaves like an ideal (lossless) capacitor of capacitance C, which is in parallel with a conductance G_P.

the capacitor in Figure 7.14, which has this dielectric medium between the plates. Then the admittance Y, *i.e.*, the reciprocal of impedance of this capacitor, with ε_r given in Equation 7.27 is

Admittance of a parallel plate capacitor

$$Y = \frac{j\omega A\varepsilon_o\varepsilon_r(\omega)}{d} = \frac{j\omega A\varepsilon_o\varepsilon_r'(\omega)}{d} + \frac{\omega A\varepsilon_o\varepsilon_r''(\omega)}{d}$$

which can be written as

$$Y = j\omega C + G_P \qquad [7.28]$$

where

$$C = \frac{A\varepsilon_o\varepsilon_r'}{d} \qquad [7.29]$$

Equivalent ideal capacitance

and

$$G_P = \frac{\omega A\varepsilon_o\varepsilon_r''}{d} \qquad [7.30]$$

Equivalent parallel conductance

is a real number just as if we had a conductive medium with some conductance G_P or resistance $1/G_P$. The admittance of the dielectric medium according to Equation 7.28 is a parallel combination of an ideal, or lossless, capacitor C, with a relative permittivity ε_r', and a resistance of $R_P = 1/G_P$ as indicated in Figure 7.14. Thus the dielectric medium behaves as if C_o and R_P were in parallel. There is no real electric power dissipated in C, but there is indeed real power dissipated in R_P because

$$\text{Input power} = IV = YV^2 = j\omega CV^2 + \frac{V^2}{R_P}$$

and the second term is real. Thus the power dissipated in the dielectric medium is related to ε_r'' and peaks when $\omega = 1/\tau$. The rate of energy storage by the field is determined by ω whereas the rate of energy transfer to molecular collisions is determined by $1/\tau$. When $\omega = 1/\tau$, the two processes, energy storage by the field and energy transfer to random collisions, are then occurring at the same rate, and hence energy is being transferred to heat most efficiently. The peak in ε_r'' versus ω is called a **relaxation peak,** which is at a frequency when the dipole relaxations are at the right rate for maximum power dissipation. This process is known as **dielectric resonance.**

According to Equation 7.28, the magnitude of G_P and hence the energy loss is determined by ε_r''. In engineering applications of dielectrics in capacitors, we would like to minimize ε_r'' for a given ε_r'. We define the relative magnitude of ε_r'' with respect to ε_r' through a quantity, $\tan\delta$, called the **loss tangent** (or **loss factor**), as

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} \qquad [7.31]$$

Loss tangent

which is frequency dependent and peaks just beyond $\omega = 1/\tau$. The actual value of $1/\tau$ depends on the material, but typically for liquid and solid media it is in the gigahertz range, that is, microwave frequencies. We can easily find the energy per unit time—power—dissipated as dielectric loss in the medium. The resistance R_P represents the dielectric loss, so

$$W_{\text{vol}} = \frac{\text{Power loss}}{\text{Volume}} = \frac{V^2}{R_P} \times \frac{1}{dA} = \frac{V^2}{\dfrac{d}{\omega A\varepsilon_o\varepsilon_r''}} \times \frac{1}{dA} = \frac{V^2}{d^2}\omega\varepsilon_o\varepsilon_r''$$

Using Equation 7.31 and $\mathcal{E} = V/d$, we obtain

$$W_{\text{vol}} = \omega\mathcal{E}^2\varepsilon_o\varepsilon_r'\tan\delta \qquad [7.32]$$

Dielectric loss per unit volume

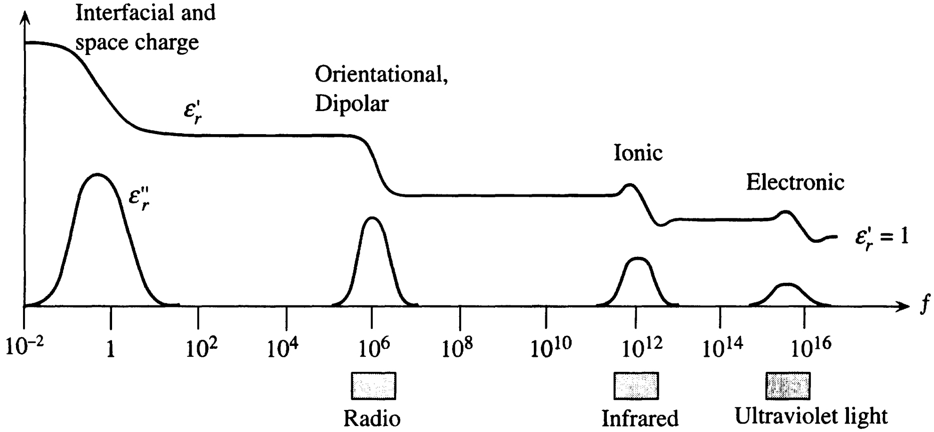
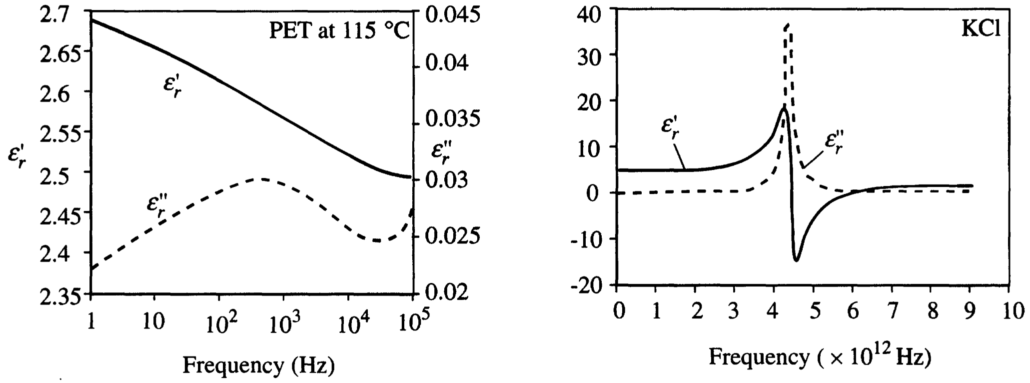


Figure 7.15 The frequency dependence of the real and imaginary parts of the dielectric constant in the presence of interfacial, orientational, ionic, and electronic polarization mechanisms.

Equation 7.32 represents the power dissipated per unit volume in the polarization mechanism: energy lost per unit time to random molecular collisions as heat. It is clear that dielectric loss is influenced by three factors: ω, $\mathcal{E}$, and $\tan \delta$.

Although we considered only orientational polarization, in general a dielectric medium will also exhibit other polarization mechanisms and certainly electronic polarization since there will always be electron clouds around individual atoms, or electrons in covalent bonds. If we were to consider the ionic polarizability in ionic solids, we would also find α_I to be frequency dependent and a complex number. In this case, lattice vibrations in the crystal, typically at frequencies ω_I in the infrared region of the electromagnetic spectrum, will dissipate the energy stored in the induced dipole moments just as energy was dissipated by molecular collisions in the gaseous dipolar medium. Thus, the energy loss will be greatest when the frequency of the polarizing field is the same as the lattice vibration frequency, $\omega = \omega_I$, which tries to randomize the polarization.

We can represent the general features of the frequency dependence of the real and imaginary parts of the dielectric constant as in Figure 7.15. Although the figure shows distinctive peaks in ε_r'' and transition features in ε_r', in reality these peaks and various features are broader. First, there is no single well-defined lattice vibration frequency but instead an allowed range of frequencies just as in solids where there is an allowed range of energies for the electron. Moreover, the polarization effects depend on the crystal orientation. In the case of polycrystalline materials, various peaks in different directions overlap to exhibit a broadened overall peak. At low frequencies the interfacial or space charge polarization features are even broader because there can be a number of conduction mechanisms (different species of charge carriers and different carrier mobilities) for the charges to accumulate at interfaces, each having its own speed. Orientational polarization, especially in many liquid dielectrics at room temperature, typically takes place at radio to microwave frequencies. In some polymeric materials, this type of polarization involves a limited rotation of dipolar side groups



(a)

(b)

Figure 7.16 Real and imaginary parts of the dielectric constant, ε'_r and ε''_r, versus frequency for (a) a polymer, PET, at 115 °C and (b) an ionic crystal, KCl, at room temperature.

Both exhibit relaxation peaks but for different reasons.

SOURCE: Data for (a) from author's own experiments using a dielectric analyzer (DEA), (b) from C. Smart, G. R. Wilkinson, A. M. Karo, and J. R. Hardy, International Conference on Lattice Dynamics, Copenhagen, 1963, as quoted by D. H. Martin, "The Study of the Vibration of Crystal Lattices by Far Infra-Red Spectroscopy," *Advances in Physics,* 14, no. 53–56, 1965, pp. 39–100.

attached to the polymeric chain and can occur at much lower frequencies depending on the temperature. Figure 7.16 shows two typical examples of dielectric behavior, ε'_r and ε''_r as a function of frequency, for a polymer (PET) and an ionic crystal (KCl). Both exhibit loss peaks, peaks in ε''_r versus frequency, but for different reasons. The particular polymer, PET (a polyester), exhibits orientational polarization due to dipolar side groups, whereas KCl exhibits ionic polarization due to the displacement of K^+ and Cl^- ions. The frequency of the loss peak in the case of orientational polarization is highly temperature dependent. For the PET example in Figure 7.16 at 115 °C, the peak occurs at around 400 Hz, even below typical radio frequencies.

EXAMPLE 7.5

DIELECTRIC LOSS PER UNIT CAPACITANCE AND THE LOSS ANGLE δ Obtain the dielectric loss per unit capacitance in a capacitor in terms of the loss tangent. Obtain the phase difference between the current through the capacitor and that through R_P. What is the significance of δ?

SOLUTION

We consider the equivalent circuit in Figure 7.14. The power loss in the capacitor is due to R_P. If V is the rms value of the voltage across the capacitor, then the power dissipated per unit capacitance W_{cap} is

$$W_{\text{cap}} = \frac{V^2}{R_P} \times \frac{1}{C} = V^2 \frac{\omega \varepsilon_o \varepsilon''_r A}{d} \times \frac{d}{\varepsilon_o \varepsilon'_r A} = V^2 \frac{\omega \varepsilon''_r}{\varepsilon'_r}$$

or

$$W_{\text{cap}} = V^2 \omega \tan \delta$$

Table 7.3 Dielectric properties of three insulators

	$f = 60$ Hz			$f = 1$ MHz		
Material	ε_r'	**tan** δ	ω **tan** δ	ε_r'	**tan** δ	ω **tan** δ
Polycarbonate	3.17	9×10^{-4}	0.34	2.96	1×10^{-2}	6.2×10^4
Silicone rubber	3.7	2.25×10^{-2}	8.48	3.4	4×10^{-3}	2.5×10^4
Epoxy with mineral filler	5	4.7×10^{-2}	17.7	3.4	3×10^{-2}	18×10^4

As tan δ is frequency dependent and peaks at some frequency, so does the power dissipated per unit capacitance. A clear design objective would be to keep W_{cap} as small as possible. Further, for a given voltage, W_{cap} does not depend on the dielectric geometry. For a given voltage and capacitance, we therefore cannot reduce the power dissipation by simply changing the dimensions of the dielectric.

Consider the rms currents through R_P and C, I_{loss} and I_{cap} respectively, and their ratio,[7]

$$\frac{I_{\text{loss}}}{I_{\text{cap}}} = \frac{V}{R_P} \times \frac{\dfrac{1}{j\omega C}}{V} = \frac{\omega \varepsilon_o \varepsilon_r'' A}{d} \times \frac{d}{j\omega \varepsilon_o \varepsilon_r' A} = -j \tan\delta$$

As expected, the two are 90° out of phase ($-j$) and the loss current (through R_P) is a factor, $\tan\delta$, of the capacitive current (through C). The ratio of I_{cap} and the total current, $I_{\text{total}} = I_{\text{cap}} + I_{\text{loss}}$, is

$$\frac{I_{\text{cap}}}{I_{\text{total}}} = \frac{I_{\text{cap}}}{I_{\text{cap}} + I_{\text{loss}}} = \frac{1}{1 + \dfrac{I_{\text{loss}}}{I_{\text{cap}}}} = \frac{1}{1 - j\tan\delta}$$

The phase angle between I_{cap} and I_{total} is determined by the negative of the phase of the denominator term $(1 - j \tan\delta)$. Thus the phase angle between I_{cap} and I_{total} is δ, where I_{cap} leads I_{total} by δ. δ is also called the **loss angle.** When the loss angle is zero, I_{cap} and I_{total} are equal and there is no loss in the dielectric.

EXAMPLE 7.6 **DIELECTRIC LOSS PER UNIT CAPACITANCE** Consider the three dielectric materials listed in Table 7.3 with their dielectric constant ε_r' (usually simply stated as ε_r) and loss factors $\tan\delta$. At a given voltage, which dielectric will have the lowest power dissipation per unit capacitance at 60 Hz? Is this also true at 1 MHz?

SOLUTION

The power dissipated at a given voltage per unit capacitance depends only on $\omega \tan\delta$, so we do not need to use ε_r'. Calculating $\omega \tan\delta$ or $(2\pi f) \tan\delta$, we find the values listed in the table at 60 Hz and 1 MHz. At 60 Hz, polycarbonate has the lowest power dissipation per unit capacitance, but at 1 MHz it is silicone rubber.

---

[7] These currents are phasors, each with a rms magnitude and phase angle.

Table 7.4 Dielectric loss per unit volume for two insulators (κ is the thermal conductivity)

	$f = 60$ Hz			$f = 1$ MHz			
Material	ε_r'	**tan** δ	**Loss (mW cm^{-3})**	ε_r'	**tan** δ	**Loss (W cm^{-3})**	κ **(W cm^{-1} K^{-1})**
XLPE	2.3	3×10^{-4}	0.230	2.3	4×10^{-4}	5.12	0.005
Alumina	8.5	1×10^{-3}	2.84	8.5	1×10^{-3}	47.3	0.33

EXAMPLE 7.7

DIELECTRIC LOSS AND FREQUENCY Calculate the heat generated per second due to dielectric loss per cm^3 of cross-linked polyethylene, XLPE (typical power cable insulator), and alumina, Al_2O_3 (typical substrate in thin- and thick-film electronics), at 60 Hz and 1 MHz at a field of 100 kV cm^{-1}. Their properties are given in Table 7.4. What is your conclusion?

SOLUTION

The power dissipated per unit volume is

$$W_{\text{vol}} = (2\pi f)\mathcal{E}^2\varepsilon_o\varepsilon_r' \tan\delta$$

We can calculate W_{vol} by substituting the properties of individual dielectrics at the given frequency f. For example, for XLPE at 60 Hz,

$$\begin{aligned} W_{\text{vol}} &= (2\pi 60\ \text{Hz})(100 \times 10^3 \times 10^2\ \text{V m}^{-1})^2(8.85 \times 10^{-12}\ \text{F m}^{-1})(2.3)(3 \times 10^{-4}) \\ &= 230\ \text{W m}^{-3} \end{aligned}$$

We can convert this into per cm^3 by

$$W_{\text{vol}}' = \frac{W_{\text{vol}}}{10^6} = 0.230\ \text{mW cm}^{-3}$$

which is shown in Table 7.4.

From similar calculations we can obtain the heat generated per second per cm^3 as shown in Table 7.4. The heats at 60 Hz are small. The thermal conductivity of the insulation and its connecting electrodes can remove the heat without substantially increasing the temperature of the insulation. At 1 MHz, the heats generated are not trivial. One has to remove 5.12 W of heat from 1 cm^3 of XLPE and 47.3 W from 1 cm^3 of alumina. The thermal conductivity κ of XLPE is about 0.005 W cm^{-1} K^{-1}, whereas that of alumina is almost 100 times larger, 0.33 W cm^{-1} K^{-1}. The poor thermal conductivity of polyethylene means that 5.12 W of heat cannot be conducted away easily and it will raise the temperature of the insulation until dielectric breakdown ensues. In the case of alumina, 47.3 W of heat will substantially increase the temperature. *Dielectric loss is the mechanism by which microwave ovens heat food.* Dielectric heating at high frequencies is used in industrial applications such as heating plastics and drying wood.

7.4.2 DEBYE EQUATIONS, COLE–COLE PLOTS, AND EQUIVALENT SERIES CIRCUIT

Consider a dipolar dielectric in which there are both orientational and electronic polarizations, α_d and α_e, respectively, contributing to the overall polarizability. Electronic polarization α_e will be independent of frequency over the typical frequency range of