Bachelor of Science with Honours in Physics
Course no.: PHY 3103 | Course title: Quantum Mechanics I
1st semester of 3rd year, Academic session: 2023–2024

Supplementary Practice Questions

1. Is $\psi(x) = Axe^{-x^2}$ an acceptable wavefunction in quantum mechanics? Justify your answer and normalize it if valid.

2. Is the following function an acceptable wavefunction in quantum mechanics? Justify your answer, and normalize it if valid.

$$\psi(x) = \begin{cases} A\sin(\pi x), & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

3. Show that eigenfunctions of a Hermitian operator corresponding to different eigenvalues are orthogonal.

4. Show that two operators $\hat{A}$ and $\hat{B}$ must commute in order to have a complete set of simultaneous eigenfunctions.

5. Show that: (a) $(\hat{A} + \hat{B})^\dagger = \hat{A}^\dagger + \hat{B}^\dagger$, (b) $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$.

6. The operator for the z-component of angular momentum in spherical coordinates is given by: $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$, where ϕ is the azimuthal angle. Suppose a wave function depends on ϕ as $\psi(\phi) = e^{im\phi}$, where m is a constant. (a) Show that $\psi(\phi)$ is an eigenfunction of $\hat{L}_z$. (b) Find the corresponding eigenvalue of $\hat{L}_z$. (c) What condition must m satisfy for $\psi(\phi)$ to be single-valued and physically acceptable?

7. Given the wave function $\psi(x) = Ae^{-\alpha x^2}$, where A and α are constants, calculate the expectation values $\langle x \rangle$ and $\langle p \rangle$.

8. Calculate the expectation value of position, $\langle x \rangle$, for the following wave functions and interpret the physical meaning of the result.

$$\psi(x) = \frac{1}{\sqrt{2a}}, \quad -a < x < a \text{ and zero elsewhere.}$$

9. Calculate the expectation value of position, $\langle x \rangle$, for the following wave functions and interpret the physical meaning of the result.

$$\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right), \quad 0 < x < a \text{ and zero elsewhere.}$$

10. Consider the operator $\hat{A} = -i\hbar\frac{d}{dx}$. (a) Write down the eigenvalue equation for $\hat{A}$. (b) Find the eigenfunctions $\psi_a(x)$ and corresponding eigenvalues a. (c) Discuss briefly the normalization of the eigenfunctions.