Lecture notes: Optics

Interference, Diffraction, and Polarization

Dr. Mohammad Abdur Rashid
November 22, 2026
just.edu.bd/t/rashid

Contents

1 Introduction: What Is Light? 2

1.1 The Corpuscular Model vs. The Wave Model 2

2 Two-Beam Interference by Division of Wavefront 3

2.1 Definition and Conditions for Interference 3

2.2 Coherence of Light 4

2.3 Young's Double-Slit Experiment 4

2.4 Resultant Intensity Distribution 6

3 Diffraction of Light 10

3.1 Definition, Types, and Physical Intuition 10

3.2 Single-Slit Intensity Distribution 10

3.3 The Diffraction Grating and X-Ray Diffraction 13

4 Polarization and Double Refraction 17

4.1 Types and Methods of Polarization 20

4.2 Producing Different Types of Polarized Light from Superposition 21

4.3 Producing Elliptically Polarized Light: Detailed Analysis 26

4.4 Malus' Law 29

4.5 Brewster's Law (Polarization by Reflection) 31

4.6 Optical Activity 33

1 Introduction: What Is Light?

Physical optics investigates the wave behavior of light and accounts for optical phenomena that cannot be explained by geometric ray tracing alone. Everyday and laboratory applications—including thin-film interference colors on soap bubbles, anti-glare polarizing filters, X-ray crystallography for determining molecular structures, and polarimetry for analyzing chiral compounds—depend directly on three fundamental wave phenomena: **interference**, **diffraction**, and **polarization**.

As noted by the optical historian Vasco Ronchi in *On The Nature of Light*, inquiry into the physical nature of light predates both Isaac Newton and Christiaan Huygens. Historically, two primary theoretical frameworks were formulated to describe optical propagation.

1.1 The Corpuscular Model vs. The Wave Model

Definition (Corpuscular Model): Proposed by Isaac Newton, the *corpuscular model* describes light as a stream of tiny, discrete, indivisible particles called *corpuscles* emitted by luminous sources and traveling in straight lines at extremely high speeds.

Definition (Wave Model): Based on Huygens' Principle (1690) and unified by James Clerk Maxwell's Electromagnetic Theory (1864), the *wave model* describes light as a transverse electromagnetic wave consisting of oscillating electric and magnetic fields propagating through space at the speed of light.

Explanation: In Newton's corpuscular theory, light particles behave analogously to elastic spheres: striking a smooth boundary produces elastic rebound (*reflection*), entering an optically denser or rarer medium alters the speed and direction of the corpuscles (*refraction*), and rectilinear propagation accounts for sharp geometric shadows. Furthermore, Newton associated distinct colors with corpuscles of different sizes. Nevertheless, the corpuscular model fails to account for interference, diffraction around obstacles, and polarization, all of which require light to propagate as a transverse wave.

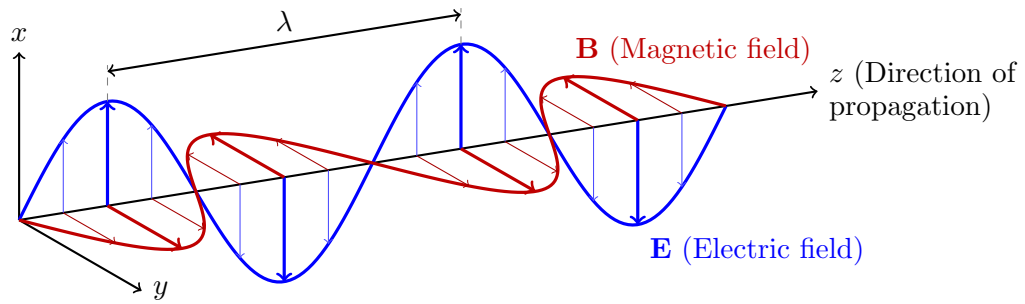


Figure 1: Schematic representation of a transverse electromagnetic wave propagating along the z-axis, illustrating the mutually perpendicular oscillations of the electric field **E** (blue) and magnetic field **B** (red).

Phenomenon	Corpuscular (Particle) Model	Wave Model (Huygens / Maxwell)
Reflection & Refraction	Explained via elastic collisions and speed changes at boundaries.	Explained via secondary wavelets (Huygens' Principle).
Sharp Shadows	Explained by straight-line motion of corpuscles.	Explained because visible wavelengths ($\sim$ 400–700 nm) are very small.
Interference	**Fails:** Particles cannot add together to produce zero light.	**Explained:** Crests and troughs superpose constructively or destructively.
Diffraction	**Fails:** Particles should not bend behind obstacles.	**Explained:** Wavefronts spread around obstacles comparable to λ.
Polarization	**Fails:** Bullet-like particles lack transverse orientation planes.	**Explained:** Light is a *transverse* electromagnetic wave.

2 Two-Beam Interference by Division of Wavefront

2.1 Definition and Conditions for Interference

Definition (Interference): *Interference* is the optical phenomenon in which two or more coherent light waves superpose in the same region of space to form a resultant wave whose amplitude and intensity depend on their relative path difference (Δ) and phase difference ($\delta = \frac{2\pi}{\lambda}\Delta$):

- **Constructive Interference (Bright Fringes):** Occurs when the crests of two waves align (*in phase*).
  - *Path Difference Condition:* $\Delta = m\lambda \quad (m = 0, \pm1, \pm2, \dots)$
  - *Phase Difference Condition:* $\delta = 2m\pi \quad (m = 0, \pm1, \pm2, \dots)$
- **Destructive Interference (Dark Fringes):** Occurs when the crest of one wave aligns with the trough of another (*out of phase by* 180°).
  - *Path Difference Condition:* $\Delta = \left(m + \frac{1}{2}\right)\lambda = (2m{+}1)\frac{\lambda}{2} \quad (m = 0, \pm1, \pm2, \dots)$
  - *Phase Difference Condition:* $\delta = (2m + 1)\pi \quad (m = 0, \pm1, \pm2, \dots)$

Explanation: Analogous to circular ripples overlapping on the surface of water, two overlapping light waves superpose algebraically at every point in space. Where two wave crests coincide ($\Delta = m\lambda$ and $\delta = 2m\pi$), the resultant displacement is reinforced; where

a crest coincides with a trough ($\Delta = (m + \frac{1}{2})\lambda$ and $\delta = (2m + 1)\pi$), the displacements cancel.

Two independent conventional light sources do not produce a stationary interference pattern because their atomic emissions undergo random phase fluctuations on timescales of approximately 10^{-10} s, washing out the interference fringes over macroscopic observation intervals. To observe a sustained, well-defined interference pattern, the superposing light waves must satisfy the following **conditions for interference**:

1. **Coherent Sources:** The waves must maintain a constant phase difference and have the same frequency.
2. **Same Wavelength / Monochromatic Light:** Using a single wavelength (λ) ensures stable, crisp fringes without color overlap.
3. **Superposition and Parallel Wavefronts:** The waves must overlap in space and time with nearly parallel wavefronts.
4. **Finite Path Difference:** The path difference must remain within the source's coherence length.

2.2 Coherence of Light

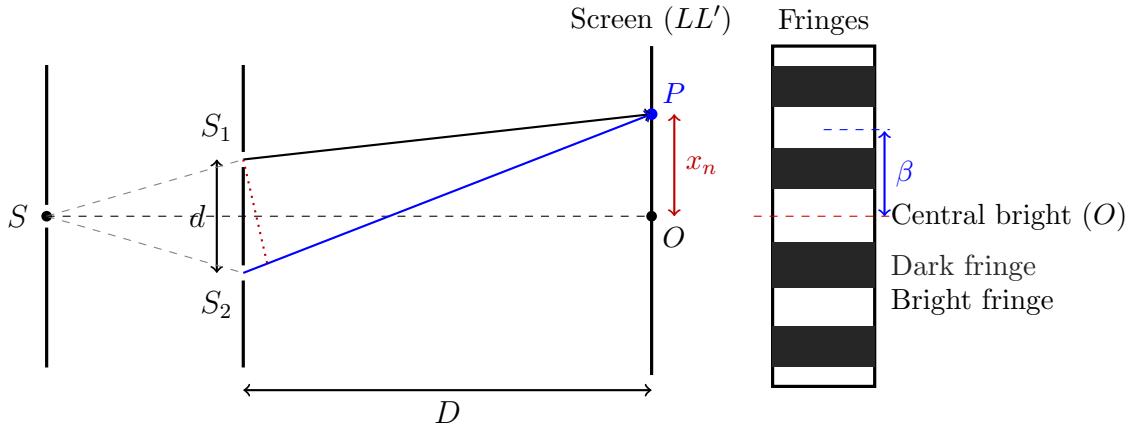
Definition (Coherence): *Coherence* refers to the property of a light wave where the phase relationship between different points on the wave is well-defined and predictable over space and time:

- **Temporal Coherence:** Consistency of phase over time at a single point. For a source of spectral bandwidth Δf, its *coherence time* (t_c) and *coherence length* (L_c) are:
$$t_c = \frac{1}{\Delta f}, \qquad L_c = c\,t_c = \frac{c}{\Delta f}$$
- **Spatial Coherence:** Uniformity of phase across different points in space perpendicular to the propagation direction.

Explanation: Lasers are *highly coherent* (both temporally and spatially), making them essential for holography and precision interferometry. LEDs, sunlight, and incandescent lamps are only *partially coherent* over very short distances, while natural thermal sources exhibit rapidly fluctuating phase relationships.

2.3 Young's Double-Slit Experiment

Definition (Young's Double-Slit Arrangement): Demonstrated by Thomas Young in 1801, this arrangement divides a single spherical wavefront from a primary slit S



(a) Ray geometry (b) Fringe pattern

Figure 2: Geometry of Young's double-slit experiment showing path difference $S_2P - S_1P$ at point P.

through two closely spaced parallel slits S_1 and S_2 (separated by distance d), producing coherent waves that interfere on a screen placed at distance D ($D \gg d$).

Step-by-Step Derivation of Fringe Positions and Fringe Width:

1. **Express the Squared Path Lengths:** Let O be the midpoint on the screen directly opposite the midpoint of S_1S_2, and let P be at distance x_n from O. By the Pythagorean theorem:

$$(S_2P)^2 = D^2 + \left(x_n + \frac{d}{2}\right)^2, \qquad (S_1P)^2 = D^2 + \left(x_n - \frac{d}{2}\right)^2$$

2. **Subtract to Find the Path Difference:**

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x_n - \frac{d}{2}\right)^2\right] = 2x_nd$$

   Factoring the left side as $(S_2P - S_1P)(S_2P + S_1P) = 2x_nd$, and noting that when $D \gg d, x_n$, we approximate $S_2P + S_1P \approx 2D$:

$$S_2P - S_1P = \frac{2x_nd}{S_2P + S_1P} \approx \frac{x_nd}{D} \quad (\text{or } d\sin\theta)$$

3. **Positions of Bright and Dark Fringes:**

   - **Bright Fringes (Constructive):** Setting $S_2P - S_1P = n\lambda$:

$$d\sin\theta = m\lambda \quad \text{or} \quad x_n = \frac{n\lambda D}{d} \qquad (n = 0, 1, 2, 3, \dots)$$

   - **Dark Fringes (Destructive):** Setting $S_2P - S_1P = \left(m + \frac{1}{2}\right)\lambda$:

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad (m = 0, \pm1, \pm2, \dots)$$

4. **Fringe Width (β):** The bright and dark fringes are equally spaced, and the distance between two consecutive bright (or dark) fringes is:

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$$

2.4 Resultant Intensity Distribution

Definition (Resultant Intensity): The *resultant intensity* I at any point P on the observation screen is the time-averaged energy flux proportional to the square of the resultant electric field amplitude produced by the superposition of the waves from S_1 and S_2.

Step-by-Step Algebraic Derivation from the Electromagnetic Wave Field:

1. **Electric Fields from S_1 and S_2:** Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields produced at point P by S_1 and S_2, respectively. Because the distances S_1P and S_2P are very large compared to the slit separation $d = S_1S_2$, the two electric field vectors point in nearly the same direction (denoted by unit vector $\hat{i}$):

$$\mathbf{E}_1 = \hat{i}\, E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right), \qquad \mathbf{E}_2 = \hat{i}\, E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)$$

2. **Superposition of the Electric Fields:** By the principle of superposition, the resultant electric field at point P is:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \hat{i}\left[E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right]$$

3. **Square the Resultant Field to Express Instantaneous Intensity:** The intensity I is proportional to the square of the electric field ($I = KE^2$, where K is a proportionality constant):

$$\begin{aligned} I =& K\left[E_{01}^2 \cos^2\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02}^2 \cos^2\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right. \\ &\left. + 2E_{01}E_{02} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right] \end{aligned}$$

Using the trigonometric identity $2\cos A \cos B = \cos(A-B) + \cos(A+B)$ on the cross-term:

$$\begin{aligned} I =& K\left[E_{01}^2 \cos^2\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02}^2 \cos^2\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right. \\ &\left. + E_{01}E_{02}\left\{\cos\left[\frac{2\pi}{\lambda}(S_2P - S_1P)\right] + \cos\left[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\right]\right\}\right] \end{aligned}$$

4. **Time-Averaging over Optical Oscillations:** Because optical frequencies are extremely high ($\omega \sim 10^{15}$ rad/s), any detector records the time average over many cycles. Over a detector's response time,

$$\langle \cos^2(\omega t - \theta) \rangle = \frac{1}{2}$$

and

$$\left\langle \cos\left[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\right] \right\rangle = 0.$$

Defining the individual intensities $I_1 = \frac{1}{2}KE_{01}^2$ and $I_2 = \frac{1}{2}KE_{02}^2$ (so $KE_{01}E_{02} = 2\sqrt{I_1 I_2}$), we obtain:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where $\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$ represents the phase difference between the waves arriving at P from S_1 and S_2.

5. **Analysis of Special Cases:**

- **Case (a) Maxima and Minima:** Since $\cos\delta$ ranges between $+1$ and -1:

$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \text{at } \delta = 2n\pi \;\; (S_2P - S_1P = n\lambda)$$

$$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \quad \text{at } \delta = (2n+1)\pi \;\; \left(S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda\right)$$

When $I_1 = I_2$, $I_{\min} = 0$; in general, if $I_1 \neq I_2$, $I_{\min} \neq 0$.

- **Case (b) Incoherent Sources:** If S_1 and S_2 are illuminated by independent sources, δ varies randomly over $\sim 10^{-10}$ s. Averaging over $\sim 10^{-8}$ s gives $\langle \cos\delta \rangle = 0$, yielding $I = I_1 + I_2$ (no interference fringes).
- **Case (c) Equal Slit Intensities ($I_1 \approx I_2 = I_0$):** When S_1P and S_2P are large compared to d, $I_1 \approx I_2 = I_0$, and using $1 + \cos\delta = 2\cos^2(\delta/2)$:

$$I = 2I_0 + 2I_0\cos\delta = 4I_0\cos^2\frac{\delta}{2}$$

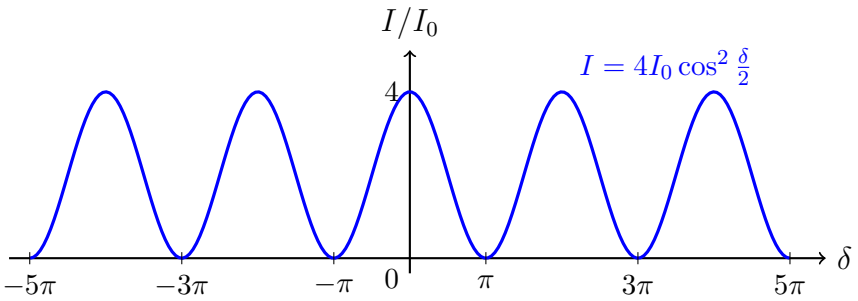


Figure 3: Variation of resultant intensity I/I_0 with phase difference δ showing the $\cos^2(\delta/2)$ fringe distribution.

Worked Example 1: Young's Double-Slit Fringe Spacing

Problem Statement: In Young's double-slit experiment, monochromatic light with a wavelength of 450 nm passes through two slits separated by a distance of 0.3 mm. The screen is placed 2.5 m away from the slits. Calculate the fringe width (β), which is the distance between two consecutive maxima, on the screen.

Step-by-Step Calculation:

1. **Convert all given quantities into SI units (meters):**

$$\lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}, \qquad d = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}, \qquad D = 2.5 \text{ m}$$

2. **Write the fringe width formula:**

$$\beta = \frac{\lambda D}{d}$$

3. **Substitute the numerical values and simplify:**

$$\beta = \frac{(450 \times 10^{-9} \text{ m})(2.5 \text{ m})}{0.3 \times 10^{-3} \text{ m}} = \frac{1125 \times 10^{-9}}{0.3 \times 10^{-3}} = 3750 \times 10^{-6} \text{ m} = 3.75 \times 10^{-3} \text{ m}$$

Final Answer: The fringe width on the screen is $\beta = 3.75$ mm.

Worked Example 2: Wavelength and Dark-Fringe Position in Double-Slit Interference

Problem Statement: In a Young's double-slit experiment, the two slits are separated by 0.25 mm and the screen is placed 1.5 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 12 mm. Calculate:

(a) the wavelength (λ) of the monochromatic light used, and

(b) the distance of the third dark fringe from the central maximum.

Step-by-Step Calculation:

1. **Convert given quantities into SI units:**

$$d = 0.25 \text{ mm} = 2.5 \times 10^{-4} \text{ m}, \qquad D = 1.5 \text{ m}, \qquad x_4 = 12 \text{ mm} = 1.2 \times 10^{-2} \text{ m}$$

2. **Part (a) — Solve for the wavelength λ using $x_n = \frac{n\lambda D}{d}$ with $n = 4$:**

$$\lambda = \frac{x_4\, d}{4\, D} = \frac{(1.2 \times 10^{-2} \text{ m})(2.5 \times 10^{-4} \text{ m})}{4 \times 1.5 \text{ m}} = \frac{3.0 \times 10^{-6}}{6.0} = 5.0 \times 10^{-7} \text{ m} = 500 \text{ nm}$$

3. **Part (b) — Calculate the position of the third dark fringe:** Dark fringes occur at $x_{\text{dark}} = \left(m + \frac{1}{2}\right) \frac{\lambda D}{d}$, where $m = 0, 1, 2, \ldots$ correspond to the first, second, and third dark fringes. Setting $m = 2$ for the third dark fringe:

$$x_{\text{dark},3} = \left(2 + \frac{1}{2}\right) \frac{\lambda D}{d} = 2.5 \times \frac{(5.0 \times 10^{-7}\ \text{m})(1.5\ \text{m})}{2.5 \times 10^{-4}\ \text{m}} = 2.5 \times 3.0 \times 10^{-3}\ \text{m} = 7.5\ \text{mm}$$

Final Answer: (a) The wavelength of light is $\lambda = 500$ nm. (b) The third dark fringe is located 7.5 mm from the central bright maximum.

Worked Example 3: Ratio of Maximum to Minimum Intensity

Problem Statement: Two coherent monochromatic light beams having an intensity ratio of $9 : 1$ superpose to form an interference pattern. Calculate:

(a) the ratio of their electric field amplitudes ($E_{01} : E_{02}$), and

(b) the ratio of the maximum intensity to the minimum intensity ($I_{\text{max}} : I_{\text{min}}$) in the resulting fringe pattern.

Step-by-Step Calculation:

1. **Part (a) — Relate intensity ratio to amplitude ratio ($I \propto E_0^2$):**

$$\frac{I_1}{I_2} = \frac{9}{1} \implies \frac{E_{01}}{E_{02}} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

2. **Part (b) — Apply the expressions for I_{max} and I_{min}:**

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2$$

3. **Substitute $\sqrt{I_1/I_2} = 3$ and simplify:**

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{3+1}{3-1}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

Final Answer: (a) The amplitude ratio is $E_{01} : E_{02} = 3 : 1$. (b) The ratio of maximum to minimum intensity is $I_{\text{max}} : I_{\text{min}} = 4 : 1$.

3 Diffraction of Light

3.1 Definition, Types, and Physical Intuition

Definition (Diffraction): *Diffraction* is the bending or spreading of waves when they encounter an obstacle, sharp edge, or narrow slit whose size is comparable to the wavelength (λ) of the wave. It is divided into two classes:

- **Fresnel Diffraction:** Occurs when the source of waves or the observation screen (or both) is at a finite distance from the obstacle.
- **Fraunhofer Diffraction:** Occurs when both the source and the observation screen are effectively at infinite distances from the slit, typically achieved using collimating and focusing lenses.

Explanation: The extent of diffraction depends directly on the ratio of the wavelength λ to the characteristic dimension of the obstacle or aperture. Acoustic waves possess wavelengths on the order of meters—comparable to doorways and structural corners—and therefore diffract readily around macroscopic obstacles. By contrast, visible light has wavelengths in the range of 400–700 nm, resulting in nearly rectilinear propagation past everyday objects. When light traverses a narrow slit or aperture whose width approaches λ, however, significant angular spreading and diffraction fringes are observed.

3.2 Single-Slit Intensity Distribution

Definition (Single-Slit Fraunhofer Diffraction): When a parallel monochromatic plane wave of wavelength λ falls normally on a long narrow slit of width b and is focused by a converging lens L (focal length f) onto a screen, wavelets from across the slit width interfere to produce a bright central maximum flanked by weaker secondary maxima separated by dark minima.

Step-by-Step Algebraic Derivation of Single-Slit Intensity:

1. **Discretize the Slit into n Point Sources:** Consider a slit of width b subdivided into n equally spaced point sources $A_1, A_2, \ldots, A_n$ with spacing Δ, so that:

   $$b = (n-1)\Delta$$

   For an incident plane wave, all points $A_1, A_2, \ldots$ are in phase. Dropping a perpendicular A_1A_2' onto the ray from A_2, the optical paths A_1B_1P and $A_2'B_2P$ are identical. Thus, the additional path traveled by the disturbance from A_2 compared to A_1 is $A_2A_2' = \Delta\sin\theta$, giving a constant phase difference ϕ between adjacent sources:

   $$\phi = \frac{2\pi}{\lambda}\Delta\sin\theta$$

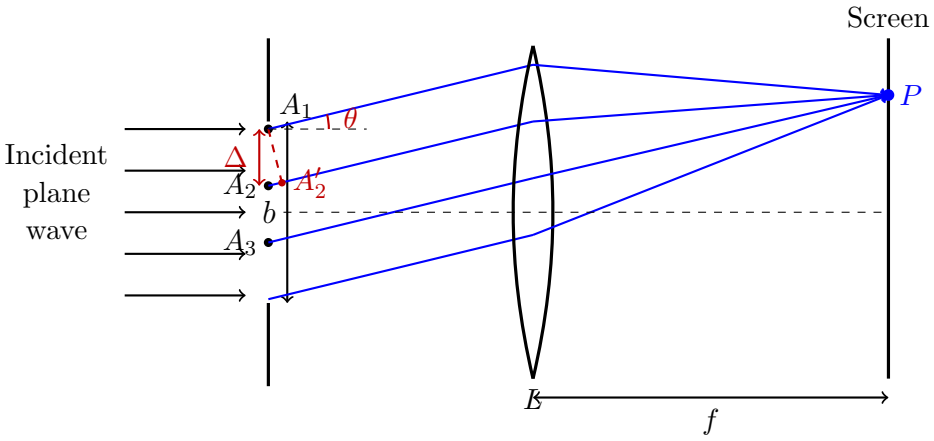


Figure 4: Geometry for single-slit Fraunhofer diffraction showing n equally spaced point sources separated by Δ, path difference $A_2A_2' = \Delta \sin\theta$, and focusing lens L.

2. **Sum the n Harmonic Disturbances at Point P:** If the field at P due to A_1 is $a\cos\omega t$, then the field from A_2 is $a\cos(\omega t - \phi)$, from A_3 is $a\cos(\omega t - 2\phi)$, and so forth. The resultant electric field at P is:

$$E = a\Big[\cos\omega t + \cos(\omega t - \phi) + \cdots + \cos\big(\omega t - (n-1)\phi\big)\Big]$$

Using the trigonometric sum identity

$$\sum_{k=0}^{n-1}\cos(\omega t - k\phi) = \frac{\sin(n\phi/2)}{\sin(\phi/2)}\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right],$$

we grt

$$E = E_\theta\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right], \qquad \text{where } E_\theta = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

3. **Take the Continuous Limit ($n\to\infty$, $\Delta\to 0$):** Letting $n\to\infty$ and $\Delta\to 0$ such that $n\Delta\to b$:

$$\frac{n\phi}{2} = \frac{\pi}{\lambda}n\Delta\sin\theta \longrightarrow \frac{\pi b\sin\theta}{\lambda} \equiv \beta$$

At the same time, $\phi = \frac{2\pi}{\lambda}\frac{b\sin\theta}{n} = \frac{2\beta}{n}\to 0$, so $\sin(\phi/2)\approx\phi/2 = \beta/n$. Substituting into E_θ:

$$E_\theta \approx \frac{a\sin(n\phi/2)}{\phi/2} = na\frac{\sin(\pi b\sin\theta/\lambda)}{(\pi b\sin\theta/\lambda)} = A\frac{\sin\beta}{\beta}$$

where $A = na$ and $\beta = \frac{\pi b\sin\theta}{\lambda}$. Thus $E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$.

4. **Resultant Intensity Distribution:** Since intensity is proportional to the square of the amplitude ($I\propto E_\theta^2$), the intensity distribution on the screen is:

$$I = I_0\frac{\sin^2\beta}{\beta^2}$$

where I_0 represents the intensity at the central maximum ($\theta = 0$, where $\beta = 0$ and $\lim_{\beta\to 0}\frac{\sin\beta}{\beta} = 1$).

5. **Positions of Minima and Secondary Maxima:**

- **Minima (Dark Fringes):** Intensity vanishes when $\sin\beta = 0$ ($\beta \neq 0$), i.e., $\beta = m\pi$:

$$b\sin\theta = m\lambda \qquad (m = \pm 1, \pm 2, \pm 3, \dots)$$

Physical interpretation: For $m = 1$ ($b\sin\theta = \lambda$), dividing the slit into two halves of width $b/2$ pairs every point in the upper half with a point $b/2$ below it having path difference $(b/2)\sin\theta = \lambda/2$ (phase difference π), causing complete destructive cancellation.

- **Maxima:** Differentiating $I(\beta)$ with respect to β and setting $dI/d\beta = 0$:

$$\frac{dI}{d\beta} = I_0\left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right) = 0 \implies \sin\beta(\beta - \tan\beta) = 0$$

Aside from $\beta = 0$ (central maximum), secondary maxima occur at the roots of the transcendental equation:

$$\tan\beta = \beta$$

Graphical intersection of $y = \beta$ and $y = \tan\beta$ gives $\beta \approx 1.43\pi$ (first secondary maximum, intensity $(\sin 1.43\pi/1.43\pi)^2 \approx 4.96\%$ of I_0), $\beta \approx 2.46\pi$ (second maximum, 1.68% of I_0), and third maximum (0.83% of I_0).

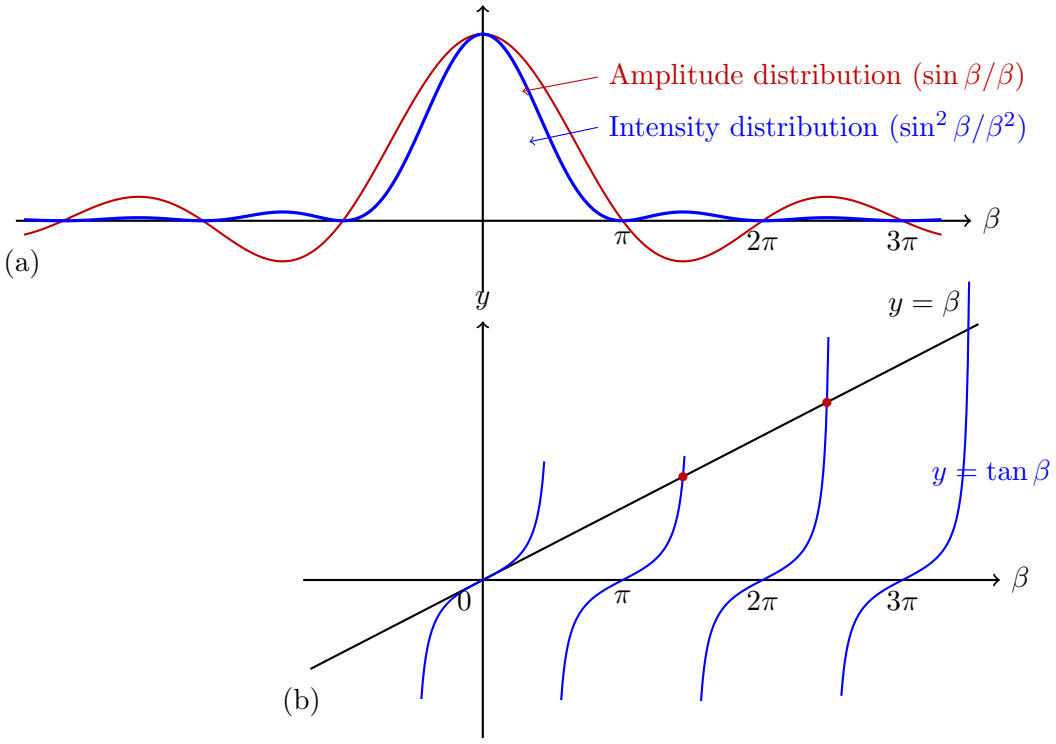


Figure 5: (a) Amplitude and intensity distributions for single-slit Fraunhofer diffraction. (b) Graphical determination of the roots of $\tan\beta = \beta$ giving the positions of secondary maxima.

3.3 The Diffraction Grating and X-Ray Diffraction

Definition (Diffraction Grating): A *diffraction grating* consists of a large number (N) of equally spaced parallel slits or grooves separated by grating spacing d (if there are N_{lines} lines per unit length, $d = 1/N_{\text{lines}}$).

Typical Grating Line Densities (N_{lines}) in Practice:

- **Teaching and Undergraduate Laboratories:** Transmission replica gratings typically contain $N_{\text{lines}} = 100$ to 600 lines/mm (1000 to 6000 lines/cm, or roughly 2500 to 15,000 lines/inch), corresponding to grating elements $d \approx 1.67$–$10\,\mu$m that produce well-separated visible-light maxima in first and second orders.
- **Analytical Spectrometers and Monochromators:** Precision ruled or holographic reflection gratings commonly use $N_{\text{lines}} = 1200$ to 3600 lines/mm (12,000 to 36,000 lines/
$d \approx 0.28$–$0.83\,\mu$m) to achieve high angular dispersion in the ultraviolet (UV) and visible regions.
- **Infrared and Echelle Spectroscopy:** Coarser gratings with $N_{\text{lines}} = 30$ to 300 lines/mm are employed for longer infrared wavelengths and high-order echelle spectrographs.

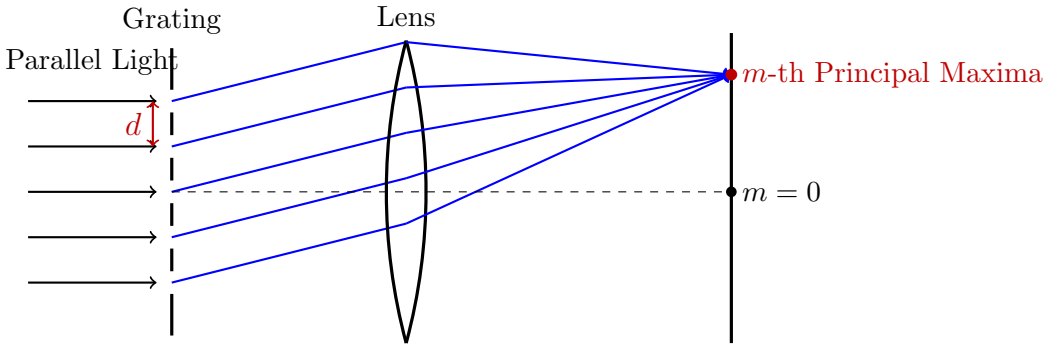


Figure 6: Multiple-slit diffraction grating producing sharp principal maxima at $d\sin\theta = m\lambda$.

Key Grating and Crystal Diffraction Equations:

- **N-Slit Intensity Distribution:** Combining single-slit diffraction ($\beta = \frac{\pi b \sin\theta}{\lambda}$) and N-slit interference ($\gamma = \frac{\pi d \sin\theta}{\lambda}$):

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

- **Grating Equation (Principal Maxima):** Intense, sharp principal maxima (intensity $\propto N^2$) occur when $\gamma = m\pi$:

$$d\sin\theta = m\lambda \qquad (m = 0, 1, 2, \dots)$$

Important Distinction: Notice that $b\sin\theta = m\lambda$ gives **minima** for a single slit, whereas $d\sin\theta = m\lambda$ gives **principal maxima** for a diffraction grating.

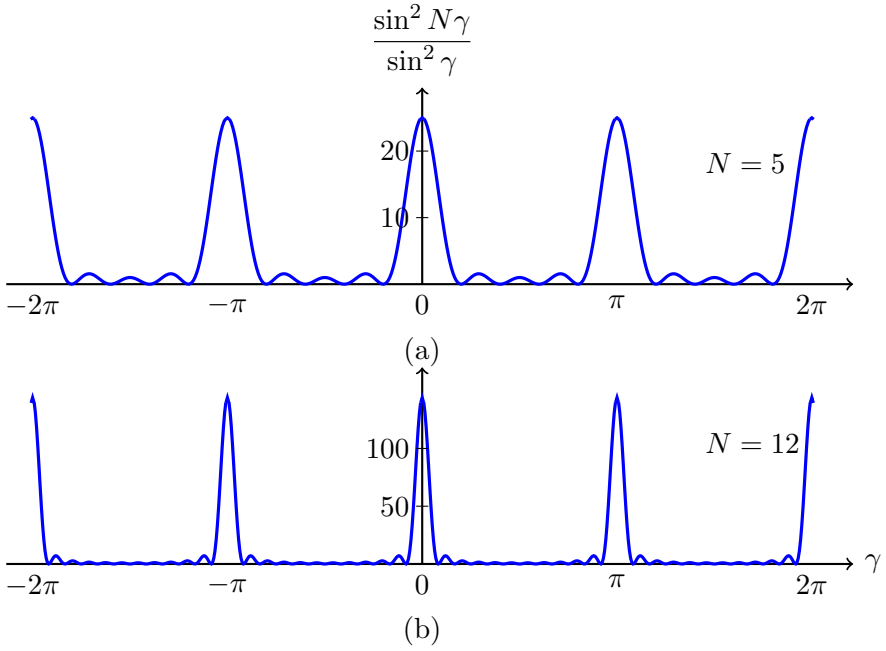


Figure 7: Variation of the N-slit interference factor $(\sin^2 N\gamma)/\sin^2\gamma$ with γ for (a) $N = 5$ and (b) $N = 12$. As N increases, the principal maxima at $\gamma = 0, \pm\pi, \pm 2\pi, \ldots$ become progressively sharper and narrower.

- **Resolving Power (R):** The ability of a grating to separate two closely spaced wavelengths differing by $\Delta\lambda$:

$$R = \frac{\lambda}{\Delta\lambda} = nN$$

  where n is the order of the spectrum and N is the total number of illuminated lines (grooves).

  - *Physical Meaning (Rayleigh Criterion):* Because each principal maximum has a finite angular width that narrows as N increases (as seen in Figure 7), two adjacent wavelengths are just resolved when the principal maximum of one falls on the first minimum of the other. Higher resolving power requires illuminating more grooves (N) or observing in a higher diffraction order (n).
  - *Practical Uses:* Resolving power determines the precision of optical spectrometers in (i) **atomic and molecular spectroscopy**, such as separating closely spaced spectral doublets like the yellow sodium D_1 (589.6 nm) and D_2 (589.0 nm) lines ($\Delta\lambda = 0.6$ nm, requiring $R \geq 982$); (ii) **astrophysics**, for analyzing stellar absorption lines, isotope shifts, and Doppler velocities; and (iii) **chemical and laser diagnostics**, for identifying overlapping emission bands and measuring narrow laser linewidths.

- **X-Ray Diffraction (XRD) — Bragg's Law:** Crystal atomic planes separated by $d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$ diffract X-rays into Laue spots according to:

$$2d_{hkl}\sin\theta = m\lambda$$

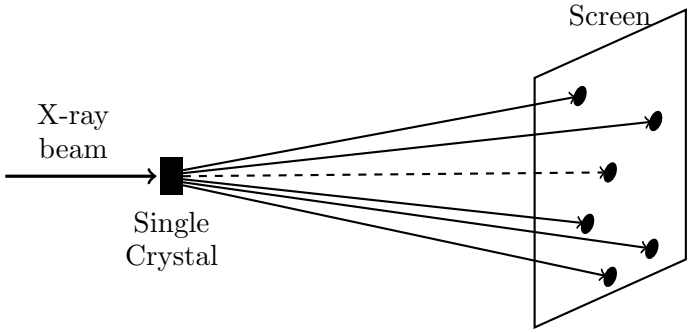


Figure 8: X-ray diffraction (Laue method): when a polychromatic X-ray beam strikes a single crystal, each set of atomic planes selects its own wavelength to satisfy Bragg's relation $2d_{hkl}\sin\theta = m\lambda$, forming discrete Laue spots.

Worked Example 4: Overlapping Spectra and Sodium D-Lines in a Grating

Problem Statement: Consider a diffraction grating with $15,000$ lines per inch.

(a) Show that if we use a white light source ($\lambda_v = 4 \times 10^{-5}$ cm for violet and $\lambda_r = 7 \times 10^{-5}$ cm for red), the second- and third-order spectra overlap.

(b) What will be the angular separation of the D_1 and D_2 lines of sodium ($\lambda \approx 6 \times 10^{-5}$ cm, $\Delta\lambda = 6 \times 10^{-8}$ cm) in the second-order spectrum?

Step-by-Step Calculation:

1. **Calculate the grating element (d):** Since 1 inch = 2.54 cm:

$$d = \frac{2.54 \text{ cm}}{15,000} = 1.69 \times 10^{-4} \text{ cm}$$

2. **Part (a) — Find diffraction angles for second ($m = 2$) and third ($m = 3$) orders:**

$$\theta_{2v} = \sin^{-1}\left(\frac{2 \times 4 \times 10^{-5}}{1.69 \times 10^{-4}}\right) = \sin^{-1}(0.473) \approx 28.2^\circ$$

$$\theta_{2r} = \sin^{-1}\left(\frac{2 \times 7 \times 10^{-5}}{1.69 \times 10^{-4}}\right) = \sin^{-1}(0.828) \approx 55.90^\circ$$

$$\theta_{3v} = \sin^{-1}\left(\frac{3 \times 4 \times 10^{-5}}{1.69 \times 10^{-4}}\right) = \sin^{-1}(0.710) \approx 45.23^\circ$$

Because $\theta_{2r}\,(55.90^\circ) > \theta_{3v}\,(45.23^\circ)$, the second- and third-order spectra overlap. (Also, since $\sin\theta_{3r} > 1$, third-order red is not observed.)

3. **Part (b) — Differentiate $d\sin\theta = m\lambda$ for small $\Delta\lambda$:**

$$(d\cos\theta)\,\Delta\theta = m\,\Delta\lambda \implies \Delta\theta = \frac{m\,\Delta\lambda}{d\left[1 - (m\lambda/d)^2\right]^{1/2}}$$

Substituting $m = 2$, $\Delta\lambda = 6 \times 10^{-8}$ cm, $\lambda = 6 \times 10^{-5}$ cm, and $d = 1.69 \times 10^{-4}$ cm:

$$\Delta\theta = \frac{2 \times 6 \times 10^{-8}}{1.69 \times 10^{-4} \left[1 - \left(\frac{2 \times 6 \times 10^{-5}}{1.69 \times 10^{-4}}\right)^2\right]^{1/2}} = 0.0010 \text{ rad} \approx 3.44 \text{ minutes}$$

Final Answer: (a) Since $\theta_{2r} > \theta_{3v}$, the second- and third-order spectra overlap. (b) The angular separation is 0.0010 rad (3.44 minutes), appearing as 34.4 minutes through a telescope of angular magnification 10.

Worked Example 5: Third-Order Diffraction Maximum

Problem Statement: A grating with 5000 lines per centimeter is illuminated by monochromatic light of wavelength 500 nm. Calculate the angular position of the third-order diffraction maximum.

Step-by-Step Calculation:

1. **Convert line density to grating spacing d in meters:**

$$5000 \text{ lines/cm} = 5 \times 10^5 \text{ lines/m} \implies d = \frac{1}{5 \times 10^5 \text{ m}^{-1}} = 2 \times 10^{-6} \text{ m}$$

2. **Apply the grating equation for $n = 3$ and $\lambda = 500$ nm $= 500 \times 10^{-9}$ m:**

$$d \sin\theta = n\lambda \implies \sin\theta = \frac{3 \times (500 \times 10^{-9} \text{ m})}{2 \times 10^{-6} \text{ m}} = \frac{1500 \times 10^{-9}}{2 \times 10^{-6}} = 0.75$$

3. **Take the inverse sine:**

$$\theta = \sin^{-1}(0.75) \approx 48.6^\circ$$

Final Answer: The third-order diffraction maximum occurs at $\theta \approx 48.6^\circ$.

Worked Example 6: Angular Separation Between First and Second Orders

Problem Statement: Calculate the angular separation between the first-order and second-order maxima in the diffraction pattern produced by a grating with $10,000$ lines per centimeter for light of wavelength 400 nm.

Step-by-Step Calculation:

1. **Convert given data to SI units:**

$$\text{Grating lines} = 10,000 \text{ lines/cm} = 10^6 \text{ lines/m} \implies d = \frac{1}{10^6} = 1 \times 10^{-6} \text{ m},$$

$$\lambda = 400 \times 10^{-9} \text{ m}$$

2. **Find the first-order angle ($n = 1$):**

$$\sin\theta_1 = \frac{1 \times (400 \times 10^{-9})}{10^{-6}} = 0.4 \implies \theta_1 = \sin^{-1}(0.4) \approx 23.6^\circ$$

3. **Find the second-order angle ($n = 2$):**

$$\sin\theta_2 = \frac{2 \times (400 \times 10^{-9})}{10^{-6}} = 0.8 \implies \theta_2 = \sin^{-1}(0.8) \approx 53.1^\circ$$

4. **Subtract to find the angular separation $\Delta\theta$:**

$$\Delta\theta = \theta_2 - \theta_1 = 53.1^\circ - 23.6^\circ = 29.5^\circ$$

Final Answer: The angular separation between the first- and second-order maxima is $\Delta\theta \approx 29.5^\circ$.

Worked Example 7: Resolving Power of a Diffraction Grating

Problem Statement: What is the resolving power of a diffraction grating if it has $10,000$ lines per centimeter and is used to resolve two wavelengths of light with a difference of 0.2 nm?

Step-by-Step Calculation:

1. **Identify the resolving power formula and given values:**

$$R = \frac{\lambda}{\Delta\lambda} = nN$$

   Here, $\Delta\lambda = 0.2$ nm. Taking a standard visible wavelength of $\lambda \approx 400$ nm:

2. **Compute R:**

$$R = \frac{400 \text{ nm}}{0.2 \text{ nm}} = 2000$$

3. **Interpret physically:** Since $R = nN$, the grating requires $nN = 2000$ illuminated lines across order n to just resolve the two spectral lines.

Final Answer: The resolving power of the grating is $R = 2000$.

4 Polarization and Double Refraction

Definition (Natural and Unpolarized Light): *Natural light* refers to light emitted by natural sources such as the Sun, stars, lightning, or fire. It is a form of *unpolarized light*, meaning that the electric field vectors of the electromagnetic waves oscillate randomly

in all possible directions perpendicular to the direction of propagation (such as sunlight, incandescent bulb light, and candlelight).

Definition (Polarized Light): *Polarized light* consists of electromagnetic waves in which the oscillations of the electric field vector are restricted to a specific, well-defined direction or deterministic path perpendicular to the direction of wave propagation (such as light transmitted through a polarizing filter, reflected at Brewster's angle, or emitted by a polarized laser).

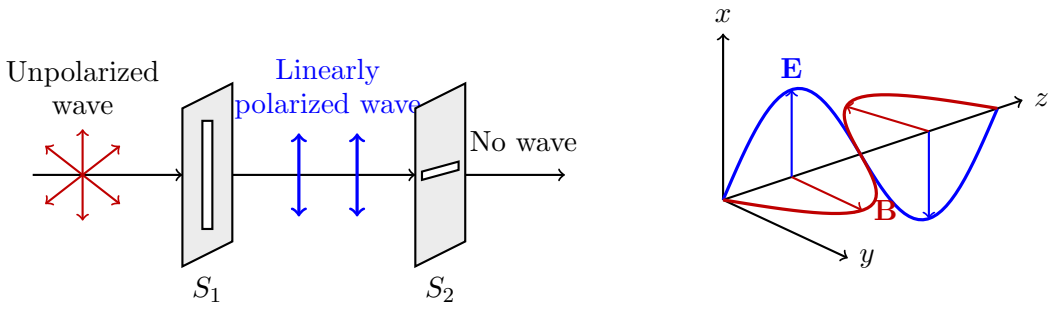
Characteristics of Natural Light:

1. **Unpolarized:** In its natural state, the electric field oscillations occur randomly in all transverse directions perpendicular to the direction of propagation.
2. **Broad Spectrum:** Natural light spans a wide range of wavelengths, encompassing visible light as well as infrared and ultraviolet radiation.
3. **Intensity and Color Dependence:** Both intensity and spectral composition vary with the source and atmospheric conditions (for example, sunlight varies with time of day and weather).
4. **Abundance:** Natural light serves as the primary source of illumination on Earth.

Characteristics of Polarized Light:

1. **Restricted Electric Field Oscillation:** The random orientation of the electric field is eliminated or reduced, confining the field vector to a single plane, circle, or ellipse in the transverse plane.
2. **Signature of Transverse Waves:** Polarization is a defining property of transverse waves; longitudinal waves cannot be polarized.
3. **Orientation-Dependent Intensity:** When passed through a second polarizer (analyzer), the transmitted intensity varies systematically with the angle between the polarization direction and the analyzer's transmission axis.
4. **Controlled Generation:** Polarized light can be produced selectively via transmission through polarizing sheets, reflection from dielectric surfaces, atmospheric scattering, or double refraction in anisotropic crystals.

Explanation: While interference and diffraction confirm that light behaves as a wave, *polarization* establishes specifically that light is a **transverse electromagnetic wave**. As illustrated in Figure 9(a), when an unpolarized transverse wave propagating along a string encounters a narrow vertical slit S_1, only the displacement component parallel to S_1 passes through, yielding a linearly polarized wave. Placing a second slit S_2 oriented perpendicular (90°) to S_1 completely blocks the wave. Figure 9(b) depicts the corresponding x-polarized electromagnetic wave propagating along the $+z$-direction, where the electric field $\mathbf{E}$ oscillates in the xz-plane ($E_x = E_0 \cos(kz - \omega t)$, $E_y = E_z = 0$)

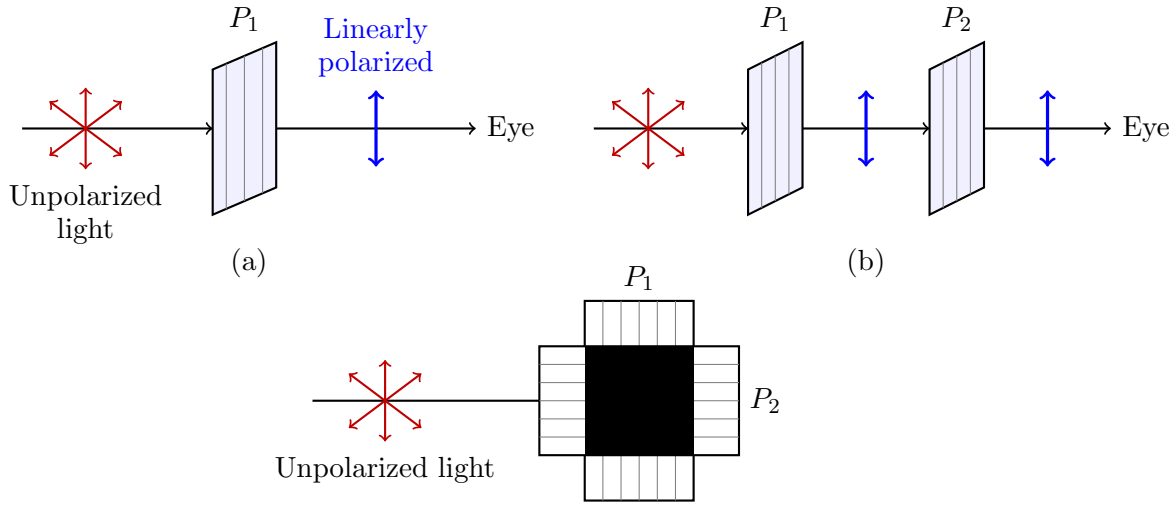


(a) (b) x-polarized EM wave

Figure 9: (a) Mechanical slit analogy demonstrating polarization by two perpendicular slits S_1 and S_2. (b) An x-polarized electromagnetic wave propagating along the z-direction with mutually perpendicular **E** and **B** oscillations.

and the orthogonal magnetic field **B** oscillates in the yz-plane ($B_y = B_0 \cos(kz - \omega t)$, $B_x = B_z = 0$).

Experimental Demonstration with Polaroid Sheets: As shown in Figure 10, when unpolarized light passes through a Polaroid sheet P_1, the emerging beam is linearly polarized along the transmission axis of P_1. Placing a second Polaroid P_2 parallel to P_1 transmits the polarized light to the observer (Figure 10b), whereas rotating P_2 by 90° (crossed Polaroids) extinguishes the light completely (Figure 10c).



(c) Crossed Polaroids (Complete Extinction)

Figure 10: (a) Linear polarization by a single Polaroid P_1; (b) full transmission through parallel Polaroids P_1 and P_2; (c) complete extinction when P_1 and P_2 are crossed at 90°.

Plane of Vibration and Plane of Polarization:

Definition (Plane of Vibration): The *plane of vibration* is the plane containing both the direction of wave propagation and the direction in which the electric field vector (**E**) oscillates.

Definition (Plane of Polarization): In classical physical optics, the *plane of polar-*

ization is defined as the plane perpendicular to the plane of vibration that contains the direction of wave propagation (and in which no electric field vibrations take place; it contains the magnetic field oscillations **B**). *(Note: In some contemporary optics texts, the plane in which the electric field vector oscillates is itself referred to as the plane of polarization.)*

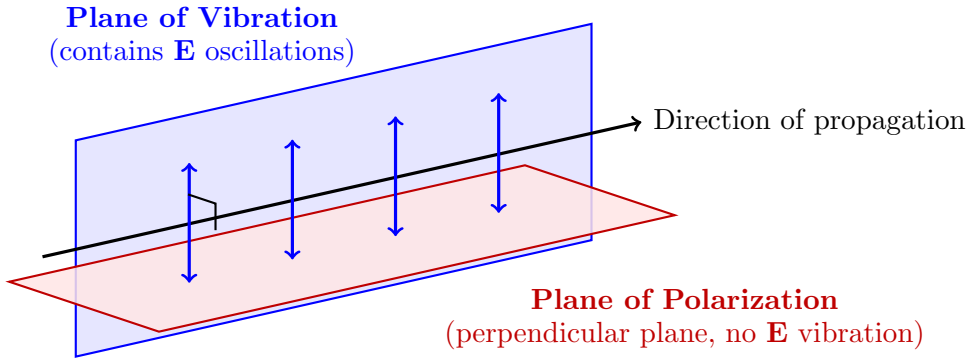


Figure 11: Geometric relationship between the *plane of vibration* (containing the electric field vector **E** and propagation direction) and the orthogonal *plane of polarization*.

4.1 Types and Methods of Polarization

Three Types of Polarization: Depending on the trajectory traced by the electric field vector in a plane perpendicular to the direction of wave propagation, polarized light is classified into three types:

1. **Linear Polarization:**
   - The electric field oscillates in only one direction perpendicular to the wave's propagation.
   - Achieved using polarizing filters or by reflection.
   - *Example:* Light passing through a Polaroid filter.
2. **Circular Polarization:**
   - The electric field vector rotates in a circular motion as the wave propagates, forming a helix.
   - Achieved using a combination of polarizers and quarter-wave plates.
   - Can be right-handed or left-handed, depending on the rotation direction.
3. **Elliptical Polarization:**
   - The electric field traces an elliptical path.
   - This is a more general form of polarization and includes linear and circular polarization as special cases.

Four Methods of Producing Linearly Polarized Light from Unpolarized Light:

1. **Polarization by Transmission:** Passing unpolarized light through a polarizing material (such as a Polaroid sheet) that transmits only the electric field component aligned with its transmission axis, yielding linearly polarized light.
2. **Polarization by Reflection:** Light reflecting off a non-metallic dielectric surface becomes partially polarized; at Brewster's angle, the reflected light is completely linearly polarized perpendicular to the plane of incidence.
3. **Polarization by Scattering:** When sunlight interacts with atmospheric particles, the scattered light becomes partially polarized.
4. **Polarization by Double Refraction (Birefringence):** Anisotropic crystals such as calcite split an unpolarized beam into two rays with orthogonal polarization states.

4.2 Producing Different Types of Polarized Light from Superposition

All states of polarization—linear, circular, and elliptical—can be generated by the superposition of two plane-polarized electromagnetic waves having specific amplitude and phase relationships.

1. Superposition of Two Collinear Plane-Polarized Waves

Consider two plane-polarized waves propagating along the z-axis with their electric field vectors oscillating along the same axis ($\hat{x}$):

$$\mathbf{E}_1 = \hat{x}\, a_1 \cos(kz - \omega t + \theta_1)$$
$$\mathbf{E}_2 = \hat{x}\, a_2 \cos(kz - \omega t + \theta_2)$$

where a_1, a_2 are the amplitudes and θ_1, θ_2 are phase constants. Applying the principle of superposition, $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, and expanding each cosine term with $\cos(A+B) = \cos A \cos B - \sin A \sin B$ (with $A = kz - \omega t$):

$$\begin{aligned}\mathbf{E} &= \hat{x}\Big[(a_1\cos\theta_1 + a_2\cos\theta_2)\cos(kz-\omega t) - (a_1\sin\theta_1 + a_2\sin\theta_2)\sin(kz-\omega t)\Big] \\ &= \hat{x}\, a\cos(kz - \omega t + \theta)\end{aligned}$$

Setting $a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2$ and $a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2$, squaring and adding both relations gives the resultant amplitude a and phase constant θ:

$$a^2 = (a_1\cos\theta_1 + a_2\cos\theta_2)^2 + (a_1\sin\theta_1 + a_2\sin\theta_2)^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)$$

$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)\right]^{1/2}$$

$$\tan\theta = \frac{a_1\sin\theta_1 + a_2\sin\theta_2}{a_1\cos\theta_1 + a_2\cos\theta_2}$$

Thus, the resultant wave remains linearly polarized along the same $\hat{x}$-axis.

2. Producing Plane-Polarized Light from Orthogonal Linearly Polarized Waves

Next, consider the superposition of two mutually perpendicular linearly polarized waves propagating along the z-axis:

$$\mathbf{E}_1 = \hat{x}\, a_1 \cos(kz - \omega t)$$

$$\mathbf{E}_2 = \hat{y}\, a_2 \cos(kz - \omega t + \theta)$$

At an arbitrary transverse plane $z = 0$ (using $\cos(-\alpha) = \cos\alpha$), the x- and y-components of the resultant electric field $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ are:

$$E_x = a_1 \cos\omega t$$

$$E_y = a_2 \cos(\omega t - \theta)$$

When the phase difference is an integer multiple of π, i.e., $\theta = n\pi$ ($n = 0, 1, 2, \dots$), expanding $\cos(\omega t - n\pi) = \cos\omega t \cos n\pi + \sin\omega t \sin n\pi = (-1)^n \cos\omega t$ yields:

$$E_x = a_1 \cos\omega t$$

$$E_y = (-1)^n a_2 \cos\omega t$$

Dividing the second equation by the first eliminates the time variable t:

$$\frac{E_y}{E_x} = \frac{(-1)^n a_2 \cos\omega t}{a_1 \cos\omega t} = \pm\frac{a_2}{a_1} \quad \text{(independent of } t\text{)}$$

where the upper sign $(+)$ corresponds to n even (in phase: $n = 0, 2, 4, \dots$) and the lower sign $(-)$ corresponds to n odd (out of phase: $n = 1, 3, 5, \dots$). Rewriting this as $E_y = \left(\pm\frac{a_2}{a_1}\right) E_x$ shows that in the E_xE_y plane, the tip of the electric vector oscillates along a straight line inclined at an angle ϕ with the E_x-axis:

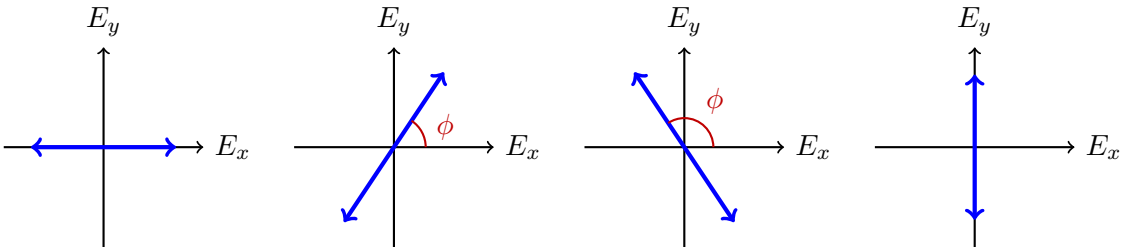
$$\tan\phi = \pm\frac{a_2}{a_1}$$

$$\phi = \tan^{-1}\left(\pm\frac{a_2}{a_1}\right)$$

Hence, the superposition of two orthogonal linearly polarized waves oscillating in phase or π out of phase produces another linearly polarized wave oriented along a new direction determined by a_2/a_1 (Figure 12).

3. Producing Circularly Polarized Light from Superposition

When $\theta \neq n\pi$, the resultant electric vector does not oscillate along a straight line. Consider two orthogonal waves of equal amplitudes ($a_1 = a_2$):



(a) $a_2 = 0$ (b) $\theta = 0$ $a_2 = 1.5a_1$ (c) $\theta = \pi$ $a_2 = 1.5a_1$ (d) $a_1 = 0$

Figure 12: Superposition of two orthogonal linearly polarized waves oscillating along the x- and y-axes for $\theta = 0$ and $\theta = \pi$, producing a resultant linearly polarized wave inclined at angle ϕ to the E_x-axis.

- **Right Circularly Polarized (RCP) Wave ($\theta = \pi/2$):** Substituting $a_2 = a_1$ and $\theta = \pi/2$ into E_x and E_y, and using $\cos(\omega t - \pi/2) = \sin \omega t$:

$$E_x = a_1 \cos \omega t$$

$$E_y = a_1 \sin \omega t$$

Squaring both equations and adding them eliminates t:

$$E_x^2 + E_y^2 = a_1^2 \cos^2 \omega t + a_1^2 \sin^2 \omega t = a_1^2(\cos^2 \omega t + \sin^2 \omega t)$$

$$E_x^2 + E_y^2 = a_1^2 \quad \text{(independent of } t\text{)}$$

For propagation in the $+z$-direction (out of the page toward an observer), evaluating (E_x, E_y) at $t = 0$ gives $(a_1, 0)$ and at $t = \frac{\pi}{2\omega}$ gives $(0, a_1)$, showing that the tip of the resultant electric vector traces a circle of radius a_1 in the **counterclockwise** direction (Figure 13c).

- **Left Circularly Polarized (LCP) Wave ($\theta = 3\pi/2$):** Substituting $a_2 = a_1$ and $\theta = 3\pi/2$, and using $\cos(\omega t - 3\pi/2) = -\sin \omega t$:

$$E_x = a_1 \cos \omega t$$

$$E_y = -a_1 \sin \omega t$$

Squaring and adding again eliminates t:

$$E_x^2 + E_y^2 = a_1^2 \cos^2 \omega t + (-a_1 \sin \omega t)^2 = a_1^2$$

At $t = 0$, $(E_x, E_y) = (a_1, 0)$, while at $t = \frac{\pi}{2\omega}$, $(E_x, E_y) = (0, -a_1)$, meaning the tip of the electric vector traces a circle of radius a_1 in the **clockwise** direction (Figure 13g).

Observer Convention and Physical Explanation of Counterclockwise Rotation in RCP Light:

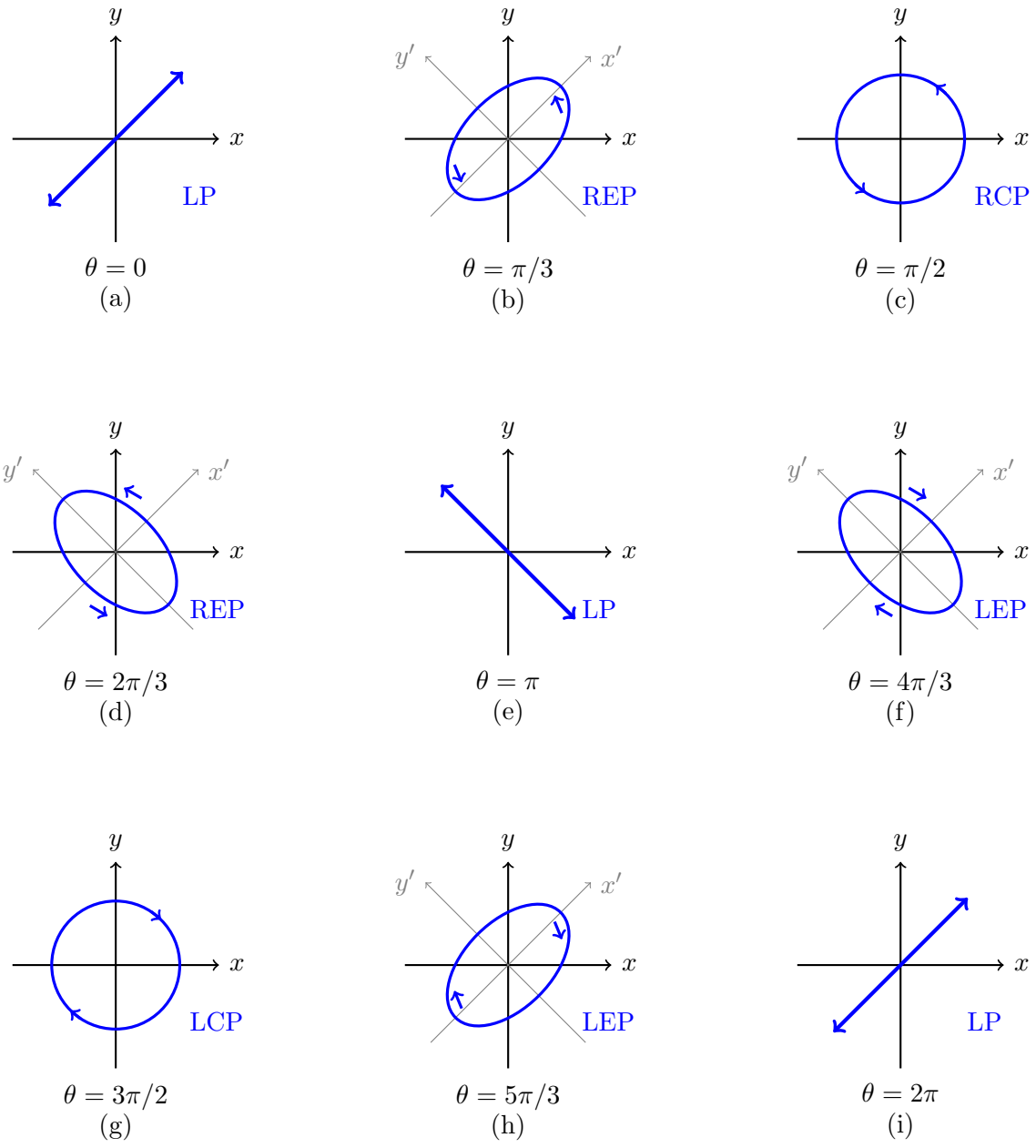


Figure 13: States of polarization for various phase differences θ corresponding to equal component amplitudes $a_1 = a_2$ (propagation is along $+z$, out of the page): linear polarization (LP), right/left circular polarization (RCP/LCP), and right/left elliptical polarization (REP/LEP).

- **Who Sees the Rotation (The Receiver / Observer Convention):** In classical optics, the sense of rotation (handedness) of circularly polarized light is always defined from the perspective of an **observer looking head-on into the oncoming wave** (i.e., facing toward the light source along the $-z$-axis while the wave propagates along $+z$ toward the observer).

- **Why the Rotation Is Counterclockwise:** Consider the RCP wave propagating along $+z$ toward the observer at a fixed observation plane $z = 0$, where $E_x = a_1 \cos \omega t$ leads $E_y = a_1 \sin \omega t$ by 90° ($\theta = \pi/2$):

  1. At $\omega t = 0$: $E_x = a_1$ and $E_y = 0$, so the resultant electric vector points along $+x$ (to the observer's right).
  2. One quarter-period later ($\omega t = \pi/2$): $E_x = 0$ and $E_y = a_1$, so the electric vector has rotated 90° upward to point along $+y$.
  3. At $\omega t = \pi$: $E_x = -a_1$ and $E_y = 0$ (pointing along $-x$, to the observer's left), and at $\omega t = 3\pi/2$, $(E_x, E_y) = (0, -a_1)$ (pointing along $-y$, downward).

  In a standard right-handed coordinate system with $+z$ pointing out of the page toward the observer, tracing $+x \to +y \to -x \to -y$ is a **counterclockwise** rotation (Figure 14). Equivalently, by the **right-hand rule**, if the fingers of the right hand curl counterclockwise in the direction of electric field rotation, the thumb points along $+z$ (the direction of propagation), which is why this state is designated *right-circularly polarized (RCP)*. (Note that if one were to look from behind the source along the direction of propagation, the same rotation would appear clockwise.)

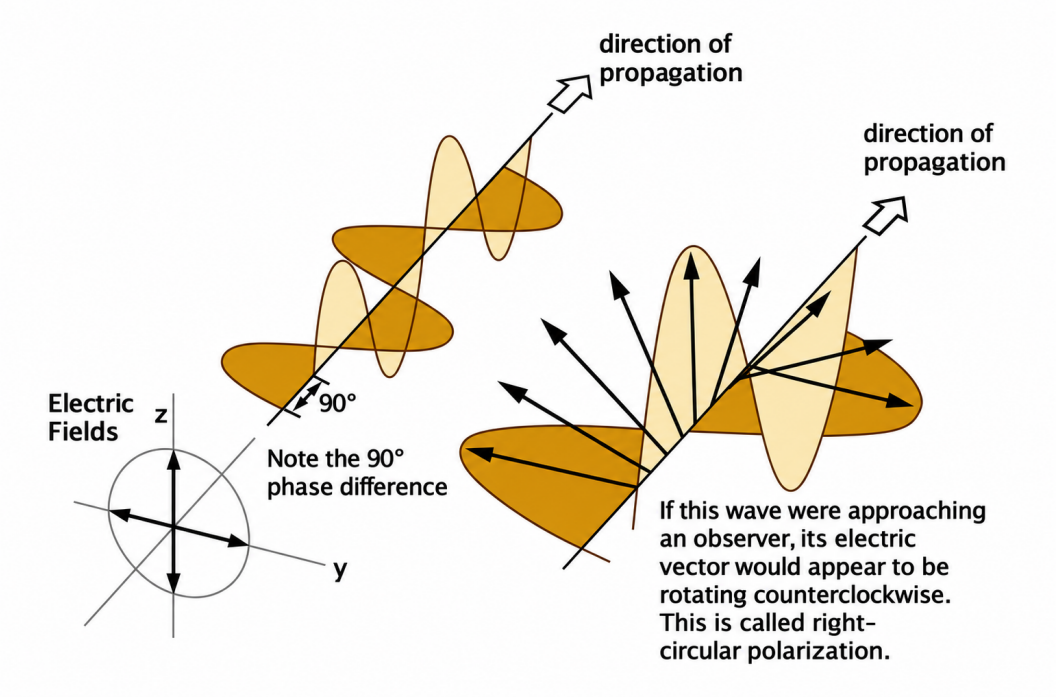


Figure 14: Formation of right-circularly polarized (RCP) light from two perpendicular electric field components with a 90° phase difference, showing the counterclockwise rotation of the resultant electric field vector as viewed by an observer facing the oncoming wave.

4. General Case: Producing Elliptically Polarized Light

For arbitrary amplitudes a_1, a_2 and phase difference $\theta \neq m\pi/2$, the tip of the electric vector rotates on an ellipse. Let the principal axes x' and y' of the ellipse be rotated by an angle α relative to the x- and y-axes:

$$E'_x = E_1 \cos(\omega t - \phi), \qquad E'_y = E_2 \sin(\omega t - \phi) \implies \left(\frac{E'_x}{E_1}\right)^2 + \left(\frac{E'_y}{E_2}\right)^2 = 1$$

Using the coordinate rotation equations $E'_x = E_x \cos\alpha + E_y \sin\alpha$ and $E'_y = -E_x \sin\alpha + E_y \cos\alpha$, substituting $E_x = a_1 \cos\omega t$ and $E_y = a_2 \cos(\omega t - \theta)$, and equating coefficients of $\cos\omega t$ and $\sin\omega t$ on both sides yields:

$$E_1 \cos\phi = a_1 \cos\alpha + a_2 \cos\theta \sin\alpha, \qquad E_1 \sin\phi = a_2 \sin\theta \sin\alpha$$

$$-E_2 \sin\phi = -a_1 \sin\alpha + a_2 \cos\theta \cos\alpha, \qquad E_2 \cos\phi = a_2 \sin\theta \cos\alpha$$

Squaring and adding all four relations confirms conservation of total intensity:

$$E_1^2 + E_2^2 = a_1^2 + a_2^2$$

Eliminating ϕ via $\frac{E_2}{E_1} = \frac{a_2 \sin\theta \cos\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \sin\alpha}$ gives the orientation angle α of the ellipse:

$$\tan 2\alpha = \frac{2a_1 a_2 \cos\theta}{a_1^2 - a_2^2}$$

Special Case ($a_1 = a_2$): When the component amplitudes are equal ($a_1 = a_2$), $\tan 2\alpha \to \infty$, giving $2\alpha = \pi/2 \implies \alpha = \pi/4$ (45°), and the semi-axis ratio simplifies to:

$$\frac{E_2}{E_1} = \frac{\sin\theta}{1 + \cos\theta} = \tan\frac{\theta}{2}$$

Evaluating $\frac{E_2}{E_1}$ across a full cycle yields:

- $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3} \implies \frac{E_2}{E_1} = +0.577, +1, +1.732$ (REP, RCP, REP — counterclockwise rotation).
- $\theta = \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3} \implies \frac{E_2}{E_1} = -1.732, -1, -0.577$ (LEP, LCP, LEP — clockwise rotation).

For example, when $\theta = 4\pi/3$ ($E_2/E_1 = -1.732$), choosing $t = 0$ such that $\phi = 0$ gives $E'_x = E_1 \cos\omega t$ and $E'_y = -1.732\, E_1 \sin\omega t$. At $t = 0$, $(E'_x, E'_y) = (E_1, 0)$; at $t = \frac{\pi}{2\omega}$, $(E'_x, E'_y) = (0, -1.732\, E_1)$; and at $t = \frac{\pi}{\omega}$, $(E'_x, E'_y) = (-E_1, 0)$, confirming clockwise rotation (LEP) with the major axis along y' (Figure 13f).

4.3 Producing Elliptically Polarized Light: Detailed Analysis

For arbitrary component amplitudes $a_1 \neq a_2$ and an arbitrary phase difference $\theta \neq n\pi$, the tip of the resultant electric vector traces an ellipse. Starting from the two orthogonal components at $z = 0$:

$$E_x = a_1 \cos\omega t$$

$$E_y = a_2 \cos(\omega t - \theta)$$

To eliminate time t, expand E_y/a_2:

$$\frac{E_y}{a_2} = \cos\omega t \cos\theta + \sin\omega t \sin\theta$$

From the first component, $\cos\omega t = \frac{E_x}{a_1}$ and $\sin\omega t = \sqrt{1 - \frac{E_x^2}{a_1^2}}$. Substituting these into the expansion gives:

$$\frac{E_y}{a_2} - \frac{E_x}{a_1}\cos\theta = \sin\theta\sqrt{1 - \frac{E_x^2}{a_1^2}}$$

Squaring both sides and rearranging terms:

$$\frac{E_y^2}{a_2^2} - \frac{2E_xE_y}{a_1a_2}\cos\theta + \frac{E_x^2}{a_1^2}\cos^2\theta = \sin^2\theta - \frac{E_x^2}{a_1^2}\sin^2\theta$$

Using $\cos^2\theta + \sin^2\theta = 1$, we obtain the general Cartesian equation of the polarization ellipse:

$$\frac{E_x^2}{a_1^2} + \frac{E_y^2}{a_2^2} - \frac{2E_xE_y}{a_1a_2}\cos\theta = \sin^2\theta$$

Because of the cross-term $-\frac{2E_xE_y}{a_1a_2}\cos\theta$, the principal axes of this ellipse are rotated relative to the x- and y-axes whenever θ is not an odd multiple of $\pi/2$.

Determination of Principal Semi-Axes (E_1, E_2) and Tilt Angle (α): Let x' and y' denote the principal axes of the ellipse, rotated by an angle α counterclockwise from the x- and y-axes. Along these principal axes, the components oscillate 90° out of phase with amplitudes E_1 and E_2:

$$E'_x = E_1\cos(\omega t - \phi)$$
$$E'_y = E_2\sin(\omega t - \phi)$$

Squaring and adding eliminates $(\omega t - \phi)$ and yields the standard canonical equation of an ellipse:

$$\left(\frac{E'_x}{E_1}\right)^2 + \left(\frac{E'_y}{E_2}\right)^2 = 1$$

Relating the rotated coordinates (E'_x, E'_y) to (E_x, E_y) via a rotation by angle α:

$$E'_x = E_x\cos\alpha + E_y\sin\alpha$$

$$E'_y = -E_x\sin\alpha + E_y\cos\alpha$$

Substituting $E_x = a_1\cos\omega t$ and $E_y = a_2(\cos\omega t\cos\theta + \sin\omega t\sin\theta)$ into the right-hand sides and expanding $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ on the left-hand sides gives:

$$E_1\cos\phi\cos\omega t + E_1\sin\phi\sin\omega t = (a_1\cos\alpha + a_2\cos\theta\sin\alpha)\cos\omega t + (a_2\sin\theta\sin\alpha)\sin\omega t$$

$$-E_2\sin\phi\cos\omega t + E_2\cos\phi\sin\omega t = (-a_1\sin\alpha + a_2\cos\theta\cos\alpha)\cos\omega t + (a_2\sin\theta\cos\alpha)\sin\omega t$$

Equating the coefficients of $\cos\omega t$ and $\sin\omega t$ independently on both sides yields four algebraic relations:

$$E_1\cos\phi = a_1\cos\alpha + a_2\cos\theta\sin\alpha$$

$$E_1 \sin\phi = a_2 \sin\theta \sin\alpha$$
$$-E_2 \sin\phi = -a_1 \sin\alpha + a_2 \cos\theta \cos\alpha$$
$$E_2 \cos\phi = a_2 \sin\theta \cos\alpha$$

a. **Conservation of Total Intensity:** Squaring all four equations and adding them together (using $\cos^2\phi + \sin^2\phi = 1$, $\cos^2\alpha + \sin^2\alpha = 1$, and $\cos^2\theta + \sin^2\theta = 1$):

$$E_1^2(\cos^2\phi+\sin^2\phi)+E_2^2(\sin^2\phi+\cos^2\phi) = a_1^2(\cos^2\alpha+\sin^2\alpha)+a_2^2(\cos^2\theta+\sin^2\theta)(\sin^2\alpha+\cos^2\alpha)$$
$$E_1^2 + E_2^2 = a_1^2 + a_2^2$$

b. **Orientation Angle (α) of the Principal Axes:** Dividing the second equation by the first, and the negative of the third equation by the fourth, gives two expressions for $\tan\phi$:

$$\tan\phi = \frac{a_2 \sin\theta \sin\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \cos\alpha}$$

Cross-multiplying these two fractions:

$$a_2^2 \sin^2\theta \sin\alpha \cos\alpha = a_1^2 \sin\alpha \cos\alpha - a_1 a_2 \cos\theta(\cos^2\alpha - \sin^2\alpha) - a_2^2 \cos^2\theta \sin\alpha \cos\alpha$$

Grouping the $a_2^2(\sin^2\theta + \cos^2\theta) = a_2^2$ terms on the left and rearranging:

$$(a_1^2 - a_2^2) \sin\alpha \cos\alpha = a_1 a_2 \cos\theta(\cos^2\alpha - \sin^2\alpha)$$

Substituting the double-angle identities $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$ and $\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$:

$$\frac{1}{2}(a_1^2 - a_2^2) \sin 2\alpha = a_1 a_2 \cos\theta \cos 2\alpha$$
$$\tan 2\alpha = \frac{2 a_1 a_2 \cos\theta}{a_1^2 - a_2^2}$$

c. **Special Case of Equal Component Amplitudes ($a_1 = a_2$):** When $a_1 = a_2$, the denominator of $\tan 2\alpha$ vanishes, so $\tan 2\alpha \to \infty$, giving $2\alpha = \pi/2$, or $\alpha = \pi/4$ ($45°$). The major and minor axes of the ellipse are therefore always inclined at $45°$ to the x- and y-axes. Setting $a_1 = a_2$ and $\alpha = 45°$ ($\sin\alpha = \cos\alpha = 1/\sqrt{2}$) in the ratio E_2/E_1:

$$\frac{E_2}{E_1} = \frac{E_2 \cos\phi}{E_1 \cos\phi} = \frac{a_1 \sin\theta \cos 45°}{a_1 \cos 45° + a_1 \cos\theta \sin 45°} = \frac{\sin\theta}{1 + \cos\theta}$$

Using the half-angle identities $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$ and $1+\cos\theta = 2\cos^2(\theta/2)$:

$$\frac{E_2}{E_1} = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)} = \tan\frac{\theta}{2}$$

Evaluating $\frac{E_2}{E_1} = \tan\frac{\theta}{2}$ across a full cycle yields:

- $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3} \implies \frac{E_2}{E_1} = +0.577, +1, +1.732$ (REP, RCP, REP — counterclockwise rotation).
- $\theta = \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3} \implies \frac{E_2}{E_1} = -1.732, -1, -0.577$ (LEP, LCP, LEP — clockwise rotation).

For example, when $\theta = 4\pi/3$ ($E_2/E_1 = -1.732$), choosing the time origin $t = 0$ such that $\phi = 0$ gives:

$$E'_x = E_1 \cos \omega t$$

$$E'_y = -1.732\, E_1 \sin \omega t$$

At $t = 0$, $(E'_x, E'_y) = (E_1, 0)$; at $t = \frac{\pi}{2\omega}$, $(E'_x, E'_y) = (0, -1.732\, E_1)$; and at $t = \frac{\pi}{\omega}$, $(E'_x, E'_y) = (-E_1, 0)$, confirming clockwise rotation (LEP) with the major axis aligned along y' (Figure 13f).

- **Right Circularly Polarized (RCP) Wave ($\theta = \pi/2$):**

  $$E_x = a_1 \cos \omega t, \qquad E_y = a_1 \sin \omega t \implies E_x^2 + E_y^2 = a_1^2 \quad \text{(independent of } t\text{)}$$

  For propagation in the $+z$-direction (out of the page), the tip of the resultant electric vector traces a circle of radius a_1 in the **counterclockwise** direction (Figure 13c).

- **Left Circularly Polarized (LCP) Wave ($\theta = 3\pi/2$):**

  $$E_x = a_1 \cos \omega t, \qquad E_y = -a_1 \sin \omega t \implies E_x^2 + E_y^2 = a_1^2 \quad \text{(independent of } t\text{)}$$

  The tip of the electric vector traces a circle of radius a_1 in the **clockwise** direction (Figure 13g).

4.4 Malus' Law

Definition (Malus' Law): When linearly polarized light of initial intensity I_0 passes through a second polarizing filter (called an *analyzer*) whose transmission axis makes an angle θ with the incident light's polarization direction, the transmitted intensity I is:

$$I = I_0 \cos^2 \theta$$

Explanation: Only the electric field component parallel to the analyzer's transmission axis ($E = E_0 \cos \theta$) passes through. Since wave intensity is proportional to the square of the electric field ($I \propto E^2$), squaring the transmitted amplitude yields $I = I_0 \cos^2 \theta$.

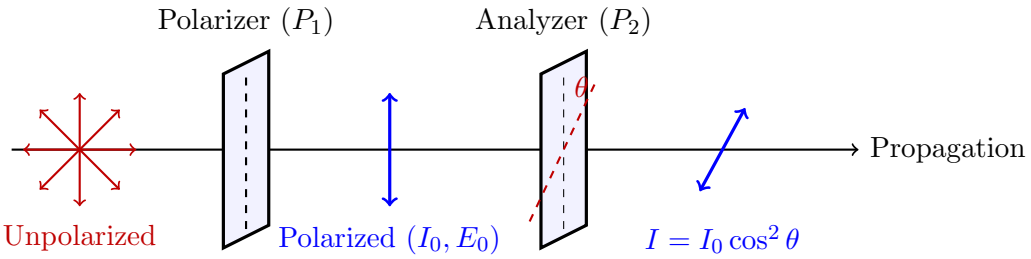


Figure 15: Transmission of linearly polarized light through an analyzer tilted at angle θ (Malus' Law).

Worked Example 8: Malus' Law at 30°

Problem Statement: A beam of light with an initial intensity of 100 W/m^2 is polarized at an angle of 30° to the axis of a polarizing filter. Calculate the transmitted intensity of the light through the polarizer.

Step-by-Step Calculation:

1. **Identify given values:** $I_0 = 100$ W/m^2, $\theta = 30°$.
2. **Apply Malus' Law:**

$$I = I_0 \cos^2\theta = 100 \times \cos^2(30°)$$

3. **Evaluate** $\cos(30°)$ **and square it:**

$$\cos(30°) = \frac{\sqrt{3}}{2} \approx 0.866 \implies \cos^2(30°) \approx (0.866)^2 \approx 0.749 \quad \text{(exact: } 0.75\text{)}$$

4. **Multiply by** I_0**:**

$$I = 100 \times 0.749 = 74.9 \text{ W/m}^2 \quad \text{(exact: } 75 \text{ W/m}^2\text{)}$$

Final Answer: The transmitted intensity is $I = 74.9$ W/m^2 (75 W/m^2).

Worked Example 9: Malus' Law at 60°

Problem Statement: A plane-polarized light of intensity 1000 W/m^2 is incident on a polarizer. If the polarizer is rotated so that the angle between the light's plane of polarization and the axis of the polarizer becomes 60°, calculate the intensity of the transmitted light.

Step-by-Step Calculation:

1. **Identify given values:** $I_0 = 1000$ W/m^2, $\theta = 60°$.

2. **Apply Malus' Law:**

$$I = I_0 \cos^2(60^\circ)$$

3. **Substitute** $\cos(60^\circ) = 0.5$**:**

$$I = 1000 \times (0.5)^2 = 1000 \times 0.25 = 250 \text{ W/m}^2$$

Final Answer: The transmitted intensity is $I = 250 \text{ W/m}^2$.

4.5 Brewster's Law (Polarization by Reflection)

Definition (Brewster's Law): *Brewster's Law* states that when unpolarized light strikes the smooth boundary of a transparent medium (such as water or glass) at a specific angle of incidence called *Brewster's angle* (or the *polarizing angle*, θ_B), the reflected light becomes **100% linearly polarized** parallel to the reflecting surface. At this exact angle, the reflected ray and the refracted ray meet at 90° ($\theta_B + \theta_r = 90^\circ$), and the tangent of Brewster's angle equals the relative refractive index (n) of the second medium with respect to the first:

$$\tan\theta_B = n = \frac{n_2}{n_1}$$

Definitions of n_1, n_2, and Their Relation to n:

- **Refractive Index of the First Medium (n_1):** The absolute refractive index of the medium through which the incident light initially travels before reaching the surface (for example, $n_1 \approx 1.00$ for air).
- **Refractive Index of the Second Medium (n_2):** The absolute refractive index of the transparent medium into which the refracted ray enters after crossing the boundary (for example, $n_2 = 1.33$ for water, or $n_2 = 1.50$ for glass).
- **Relation to Relative Refractive Index (n):** The ratio $n = \frac{n_2}{n_1}$ is the *relative refractive index* of medium 2 with respect to medium 1 (also written as ${}_1n_2$). When light travels from air ($n_1 \approx 1.00$) into another medium, $n = \frac{n_2}{1.00} = n_2$, so Brewster's Law simplifies directly to $\tan\theta_B = n$.

Simple Physical Explanation: Unpolarized light consists of electric field vibrations in all directions perpendicular to the ray: some parallel to the reflecting surface (represented by dots in Figure 16) and some in the plane of incidence (represented by double-headed arrows). When light enters the second medium at Brewster's angle, electrons at the surface oscillate along the electric field of the refracted wave. Because the reflected ray is at 90° to the refracted ray, the component of electron vibration in the plane of incidence points directly along the reflected ray. Since transverse light waves cannot vibrate along their direction of travel, no wave with plane-of-incidence polarization can be reflected—leaving the reflected beam purely polarized parallel to the surface.

Step-by-Step Derivation of Brewster's Law:

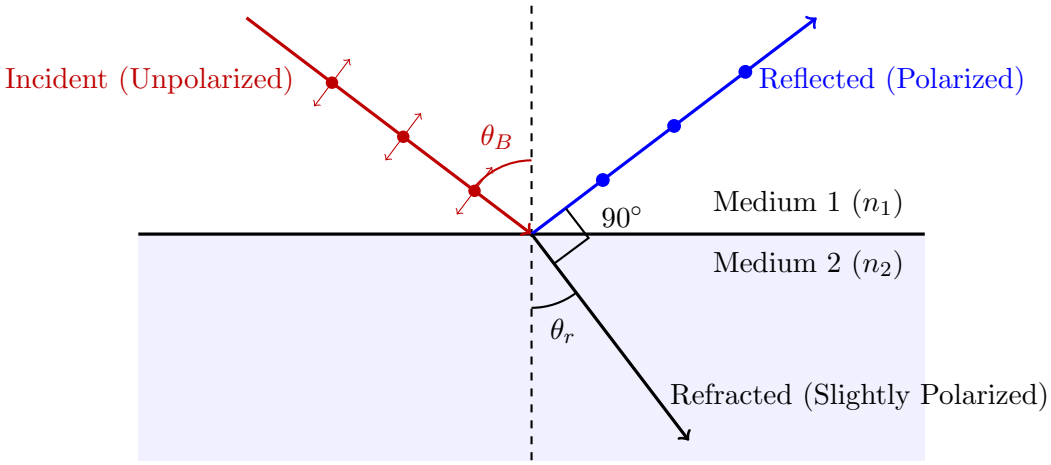


Figure 16: Polarization by reflection at Brewster's angle θ_B, where reflected and refracted rays meet at 90°.

1. **Apply Snell's Law at the boundary between medium 1 (n_1) and medium 2 (n_2):**
$$n_1 \sin\theta_B = n_2 \sin\theta_r$$

2. **Apply the 90° Condition:** At Brewster's angle, the reflected ray (at angle θ_B) and the refracted ray (at angle θ_r) are perpendicular, so $\theta_B + 90° + \theta_r = 180°$, which gives:
$$\theta_r = 90° - \theta_B$$

3. **Substitute $\theta_r = 90° - \theta_B$ into Snell's Law:** Using $\sin(90° - \theta_B) = \cos\theta_B$:
$$n_1 \sin\theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos\theta_B$$

4. **Rearrange to obtain Brewster's Law:** Dividing both sides by $n_1 \cos\theta_B$:
$$\frac{\sin\theta_B}{\cos\theta_B} = \frac{n_2}{n_1} \implies \tan\theta_B = \frac{n_2}{n_1} = n$$

What Happens When Light Travels from Air to Water? When unpolarized light (such as sunlight) travels from air ($n_1 \approx 1.00$) and strikes a calm water surface ($n_2 = 1.33$):

1. **Brewster's Angle:** The relative refractive index is $n = \frac{n_2}{n_1} = \frac{1.33}{1.00} = 1.33$, giving a polarizing angle of $\theta_B = \tan^{-1}(1.33) \approx 53.1°$ measured from the normal to the water surface.

2. **Refracted Ray Direction:** The light entering the water bends toward the normal at an angle of refraction $\theta_r = 90° - 53.1° = 36.9°$, forming an exact 90° angle with the reflected beam.

3. **Polarization State of Reflected and Refracted Beams:** At $\theta_B = 53.1°$, the reflected light bouncing off the water is **100% horizontally linearly polarized**

(parallel to the water surface), creating intense horizontal glare, whereas the refracted light entering the water remains **partially polarized** (enriched in the vertical component). At any incidence angle other than 53.1°, both the reflected and refracted beams are only partially polarized.

Applications: Polarizing sunglasses use vertical transmission axes to block horizontally polarized glare reflected off roads and water surfaces; camera polarizing filters suppress unwanted reflections from glass and water; Brewster windows produce linearly polarized laser beams without reflection loss; and measuring θ_B determines the refractive index n of unknown transparent materials.

Worked Example 10: Brewster's Angle for a Water Surface

Problem Statement: A beam of light is incident on a water surface at Brewster's angle. If the refractive index of water is 1.33, calculate the Brewster's angle at which the reflected light is fully polarized.

Step-by-Step Calculation:

1. **Identify the refractive indices:** Incident medium is air ($n_1 \approx 1.00$), and water has $n_2 = 1.33$.
2. **Apply Brewster's Law:**
$$\tan\theta_B = \frac{n_2}{n_1} = 1.33$$
3. **Solve for θ_B:**
$$\theta_B = \tan^{-1}(1.33) \approx 53.1^\circ$$

Final Answer: Reflected light from the water surface is completely polarized at $\theta_B \approx 53.1^\circ$.

4.6 Optical Activity

Definition (Optical Activity): *Optical activity* is the ability of certain *chiral* materials (molecules whose 3D structure is non-superimposable on its mirror image, such as sugars and organic enantiomers) to rotate the plane of polarization of plane-polarized light by an observed rotation angle α:

- **Dextrorotatory (+):** Rotates the plane of polarization clockwise (right-handed).
- **Levorotatory (−):** Rotates the plane of polarization counterclockwise (left-handed).

Physical Origin: Chirality and Circular Birefringence Two complementary mechanisms explain why optical rotation occurs:

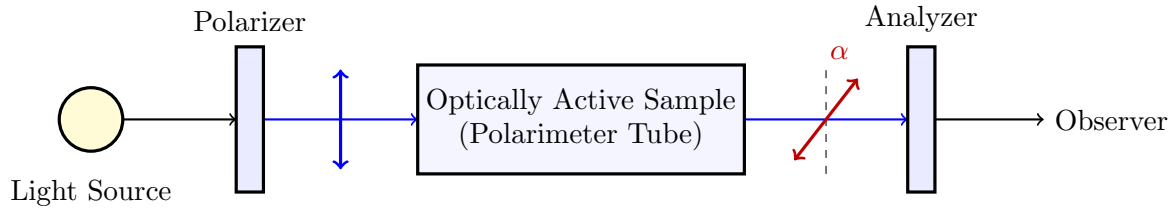


Figure 17: Schematic of a polarimeter measuring the optical rotation angle α caused by a chiral sample.

- **Molecular Chirality (Structural Origin):** For a liquid or dissolved compound to exhibit optical activity, its molecules must possess *chirality*—meaning the molecule lacks an internal plane of symmetry and cannot be superimposed on its mirror image. Common examples include sugars (such as glucose and sucrose), amino acids, and many organic pharmaceuticals. Certain crystalline solids, such as quartz, also exhibit optical activity due to a helical arrangement of atoms in their crystal lattice.
- **Fresnel's Wave Explanation (Optical Origin):** Any linearly polarized wave can be represented as the superposition of two circularly polarized waves of equal amplitude rotating in opposite directions: one *right-circularly polarized (RCP)* and one *left-circularly polarized (LCP)*. In an optically active (chiral) medium, the asymmetric molecular structure interacts differently with left- and right-handed helical fields, resulting in slightly different refractive indices for the two components ($n_R \neq n_L$). Because their speeds in the medium ($v_R = c/n_R$ and $v_L = c/n_L$) differ, one circular component advances in phase relative to the other. Upon exiting the medium, the two components recombine into a linearly polarized wave whose plane of vibration has rotated by a net angle α.

Quantitative Relation: Biot's Law and Specific Rotation Experimentally, the observed angle of rotation (α) is directly proportional to the path length (l) of light through the sample and the concentration (c) of the chiral molecules:

$$\alpha = [\alpha]_\lambda^T \cdot l \cdot c$$

where:

- α is the observed angle of optical rotation (in degrees, measured using a polarimeter).
- l is the length of the polarimeter sample tube in *decimeters* (dm, where 1 dm = 10 cm).
- c is the concentration of the optically active substance in grams per milliliter (g/mL).
- $[\alpha]_\lambda^T$ is the *specific rotation*, a characteristic constant of the substance measured at temperature T (typically 20°C) and wavelength λ (typically the sodium D-line at 589.3 nm), expressed in units of $\text{deg} \cdot \text{dm}^{-1} \cdot (\text{g/mL})^{-1}$.

Real-World Examples and Applications:

- **Sucrose and Invert Sugar (Sugar Analysis):** Aqueous sucrose (table sugar) is dextrorotatory with a specific rotation of $[\alpha]_D^{20} = +66.5°$. When sucrose undergoes acid or enzymatic hydrolysis, it splits into an equimolar mixture of D-glucose ($[\alpha]_D^{20} = +52.7°$) and D-fructose ($[\alpha]_D^{20} = -92.4°$). Because the stronger levorotation of fructose outweighs the dextrorotation of glucose, the net rotation changes sign from positive to negative—which is why the resulting mixture is called *invert sugar*. Polarimeters are routinely used in the food and sugar industries to determine sugar concentrations rapidly.
- **Pharmaceutical Enantiomers:** Mirror-image pairs of chiral drug molecules (*enantiomers*) have identical boiling points, densities, and solubilities, so standard physical tests cannot separate them. However, because biological enzymes and receptors are chiral, the two enantiomers can behave very differently in the human body—for instance, (*S*)-ibuprofen provides pain relief while (*R*)-ibuprofen is biologically inactive. Polarimetry distinguishes between enantiomers and verifies the optical purity of medications.

Worked Example 11: Determining Sugar Concentration with a Polarimeter

Problem Statement: A polarimeter tube of length 20 cm is filled with an aqueous solution of sucrose whose specific rotation is $+66.5° \text{ dm}^{-1}(\text{g/mL})^{-1}$. When monochromatic sodium light passes through the solution, the plane of polarization is rotated clockwise by $+19.95°$. Calculate the concentration of the sucrose solution in g/mL and g/L.

Step-by-Step Calculation:

1. **Convert the tube length into decimeters (dm):**

$$l = 20 \text{ cm} = \frac{20}{10} \text{ dm} = 2.0 \text{ dm}$$

2. **Identify the known quantities:**

$$\alpha = +19.95°, \qquad [\alpha]_D^{20} = +66.5° \text{ dm}^{-1}(\text{g/mL})^{-1}$$

3. **Rearrange Biot's Law ($\alpha = [\alpha] \cdot l \cdot c$) to solve for concentration c:**

$$c = \frac{\alpha}{[\alpha]_D^{20} \cdot l}$$

4. **Substitute the values and compute c:**

$$c = \frac{19.95°}{66.5° \text{ dm}^{-1}(\text{g/mL})^{-1} \times 2.0 \text{ dm}} = \frac{19.95}{133.0} \text{ g/mL} = 0.15 \text{ g/mL}$$

5. **Convert to grams per liter (g/L):**

$$c = 0.15 \text{ g/mL} \times 1000 \text{ mL/L} = 150 \text{ g/L}$$

Final Answer: The concentration of the sucrose solution is 0.15 g/mL (or 150 g/L).

Formula Summary

Topic	Key Formula	Physical Meaning of Symbols
Coherence	$t_c = \dfrac{1}{\Delta f}, \quad L_c = \dfrac{c}{\Delta f}$	t_c: coherence time; L_c: coherence length; Δf: frequency spread.
Two-Beam Intensity	$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$	$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$: phase difference; $I = 4I_0\cos^2(\delta/2)$ if $I_1 = I_2$.
Young's Double-Slit	$x_n = \dfrac{n\lambda D}{d}, \quad \beta = \dfrac{\lambda D}{d}$	x_n: n-th bright fringe; β: fringe width; d: slit spacing; D: screen distance.
Single-Slit Diffraction	$I = I_0\dfrac{\sin^2\beta}{\beta^2}, \quad b\sin\theta = m\lambda$	$\beta = \frac{\pi b\sin\theta}{\lambda}$; b: slit width; $b\sin\theta = m\lambda$ gives **minima** ($m \neq 0$).
Diffraction Grating	$d\sin\theta = m\lambda, \quad R = \dfrac{\lambda}{\Delta\lambda} = nN$	d: grating spacing; m, n: order; R: resolving power; N: illuminated lines.
Bragg's Law (XRD)	$2d_{hkl}\sin\theta = m\lambda$	d_{hkl}: interplanar crystal spacing; θ: glancing Bragg angle.
Malus' Law	$I = I_0\cos^2\theta$	I_0: incident polarized intensity; θ: angle between polarization and analyzer.
Brewster's Law	$\tan\theta_B = n = \dfrac{n_2}{n_1}$	θ_B: polarizing angle where reflected and refracted rays are at 90°.

Note: These lecture notes were prepared with the assistance of AI educational tools (Gemini) for pedagogical structuring, step-by-step algebraic typesetting, and diagram generation.