The Wave Function

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The Wave Function

The configuration or state of a quantum object is completely specified by the wave function.

The wave function of a particle, at a particular time, contains all the information that anybody at that time can have about the particle (e.g. position, momentum, and energy).

The wave function, in general, is not a real quantity, but a complex-valued functions of space and time.

The Wave Function

In one dimension the wave function is denoted by $\Psi(x,t)$, while in three dimension it is $\Psi(x,y,z,t)$.

How do we get the wave function?

The wave function is derived by solving Schrödinger equation.

The Wave Function

The Schrödinger equation of the particle moving in the potential $V(x)$ is written as

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2}+V(x)\Psi(x)=E\Psi(x)$$

The Wave Function

The wave function itself has no physical interpretation.
It is not measurable.

The square of the absolute value of the wave function has a physical interpretation.

In one dimension, we interpret $|\Psi(x,t)|^2$ as a probability density, a probability per unit length of finding the particle at a time *t* at position x.

The Wave Function

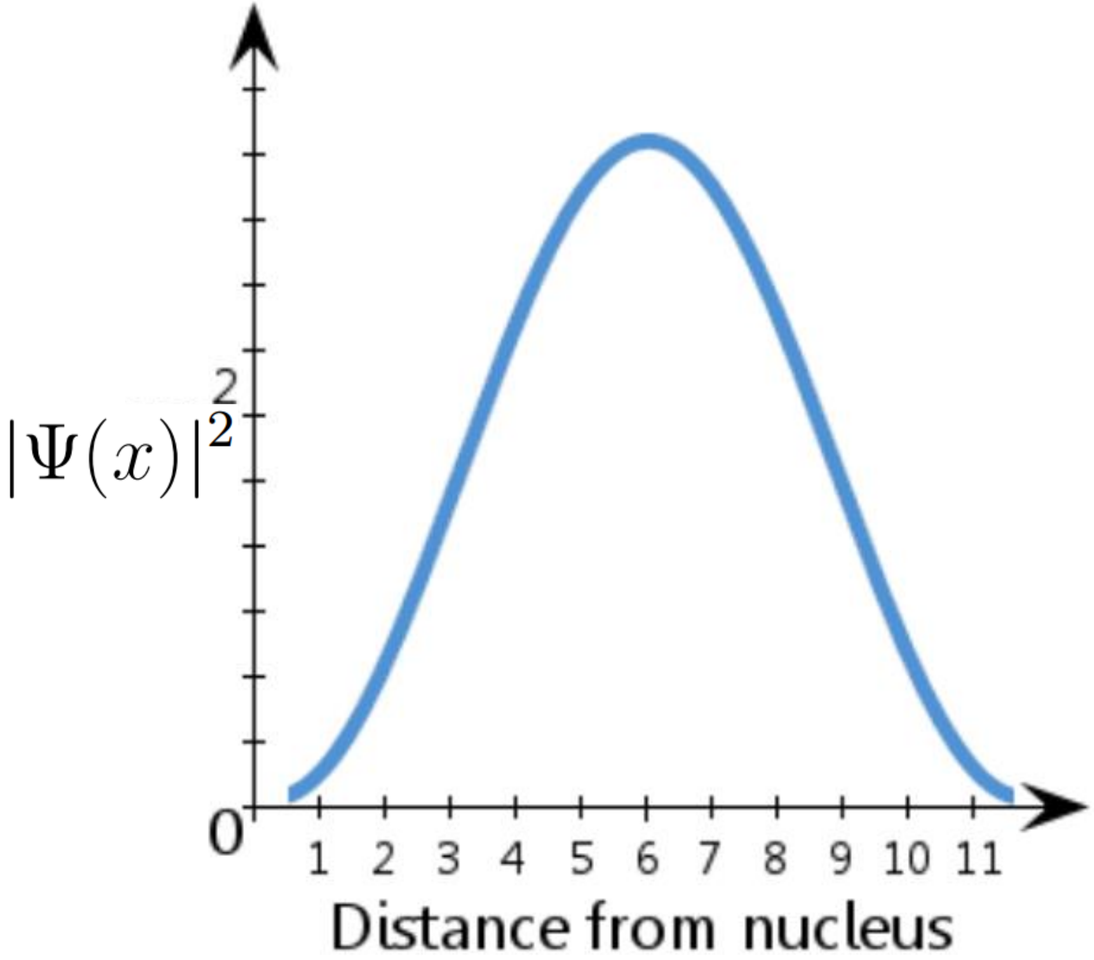
Since the wave function, in general, is a complex function we write

$$\boxed{|\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t)}$$

$$|\Psi(x,t)|^2 \mathrm{d}x = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } x \text{ and } (x+\mathrm{d}x) \text{, at time } t \end{array} \right\}$$

The Wave Function

A (fictitious) schematic diagram showing $|\Psi(x)|^2$ against distance from nucleus, in arbitrary unit.



There is a 100% probability that the electron is somewhere - in other words a probability of 1.

The Wave Function

The normalization condition of the wave function

$$\boxed{\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \mathrm{d}x = 1}$$

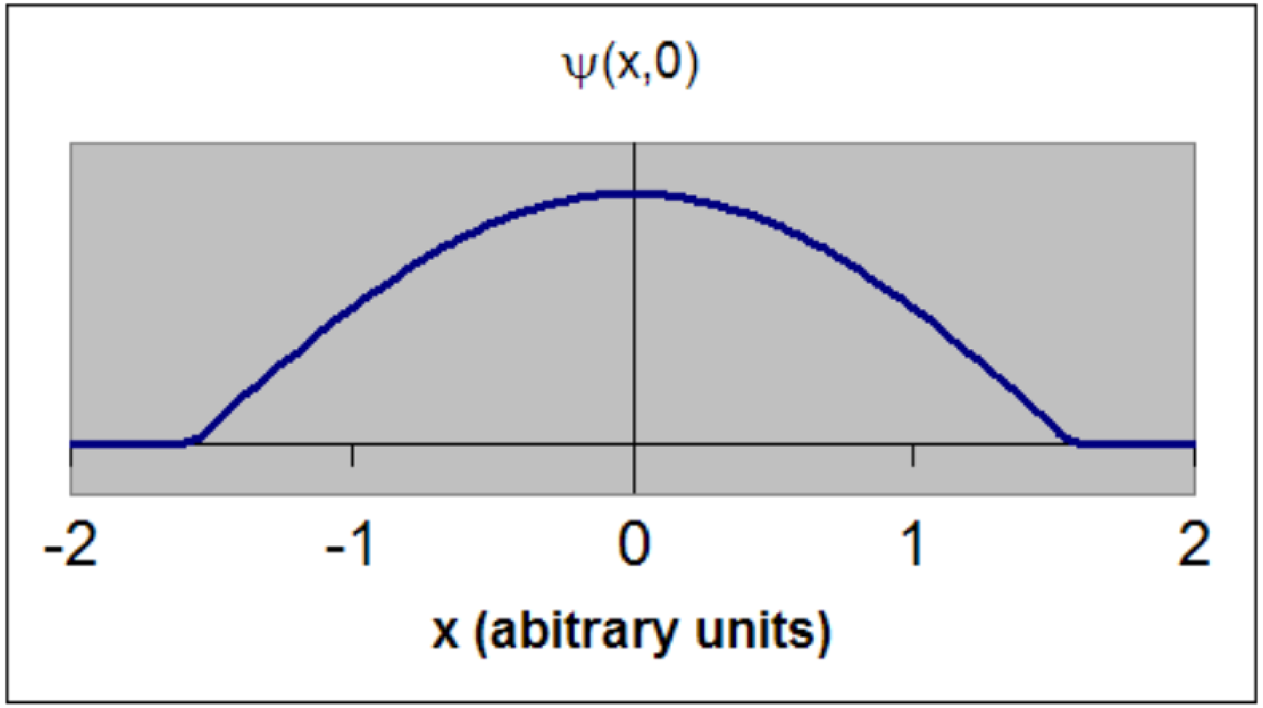
The wave function must be single valued and continuous.

The Wave Function

Single-valued wave function: $\Psi(x) = x^2$

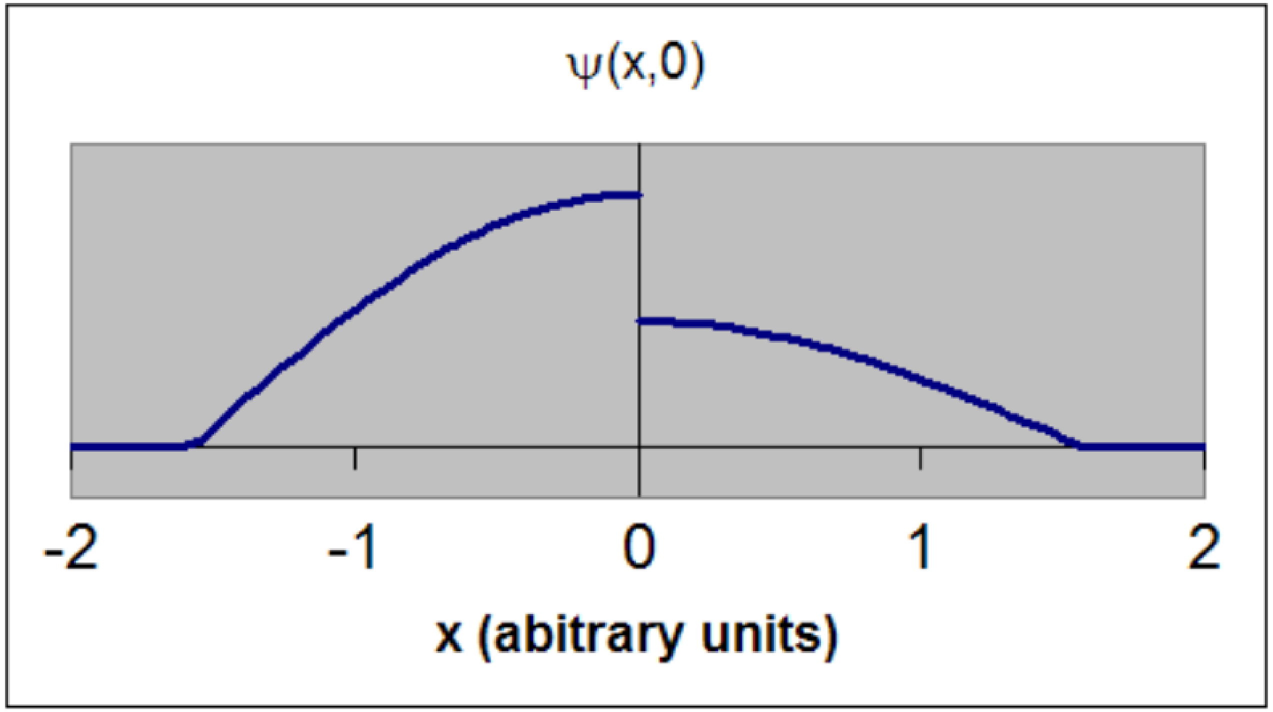
Multi-valued wave function: $\Psi(x) = \pm\sqrt{x}$

The Wave Function



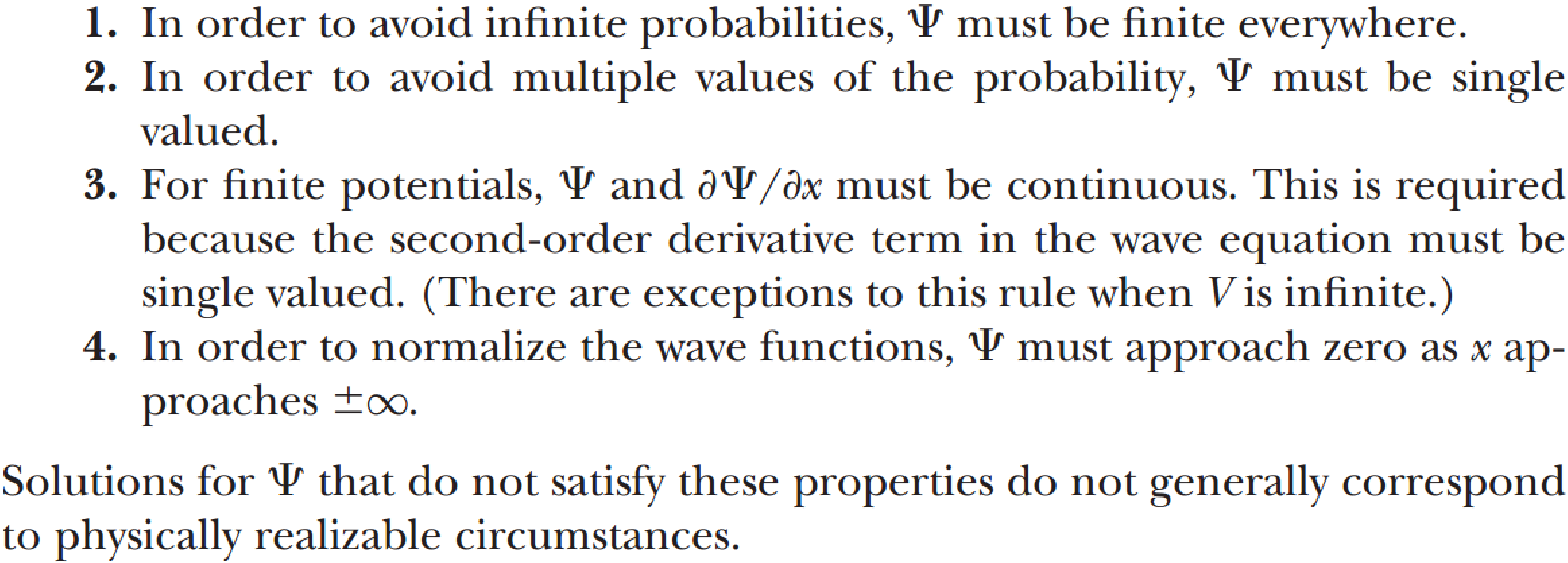
This is an acceptable wave function.
It is single valued and continuous.

The Wave Function



This is an unacceptable wave function. $\Psi(x,0)$ does not have a unique single value at $x = 0$. The wave function is not continuous.

The Wave Function



1. In order to avoid infinite probabilities, Ψ must be finite everywhere.
2. In order to avoid multiple values of the probability, Ψ must be single valued.
3. For finite potentials, Ψ and $\partial\Psi/\partial x$ must be continuous. This is required because the second-order derivative term in the wave equation must be single valued. (There are exceptions to this rule when V is infinite.)
4. In order to normalize the wave functions, Ψ must approach zero as x approaches $\pm\infty$.

Solutions for Ψ that do not satisfy these properties do not generally correspond to physically realizable circumstances.

Thank You

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