Infinite Square-Well Potential

Particle in a Box in 1D

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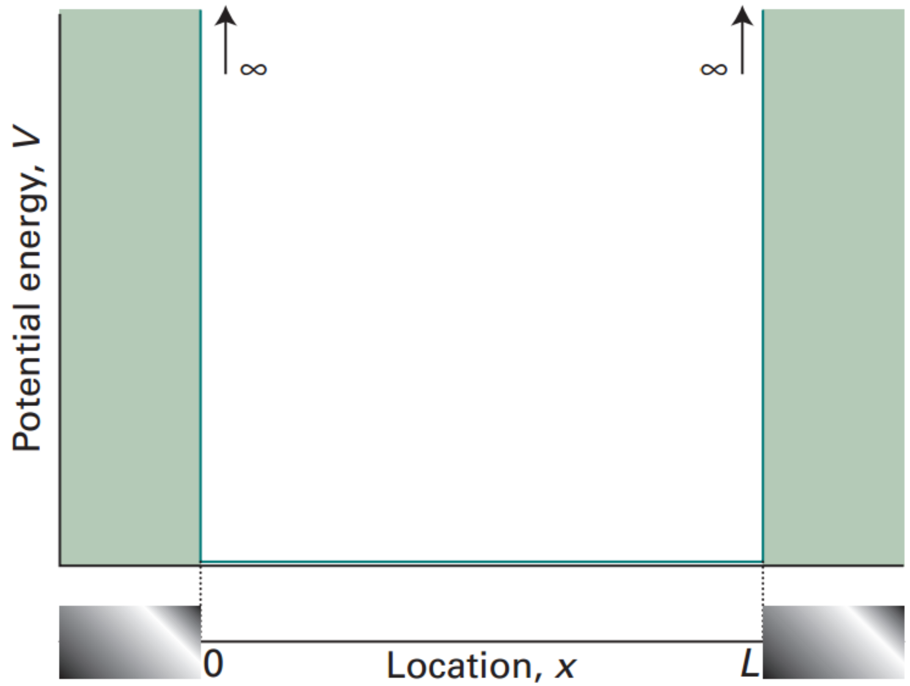
Infinite Square-Well Potential

$$V(x) = \begin{cases} \infty & x \leq 0,\ x \geq L \\ 0 & 0 < x < L \end{cases}$$



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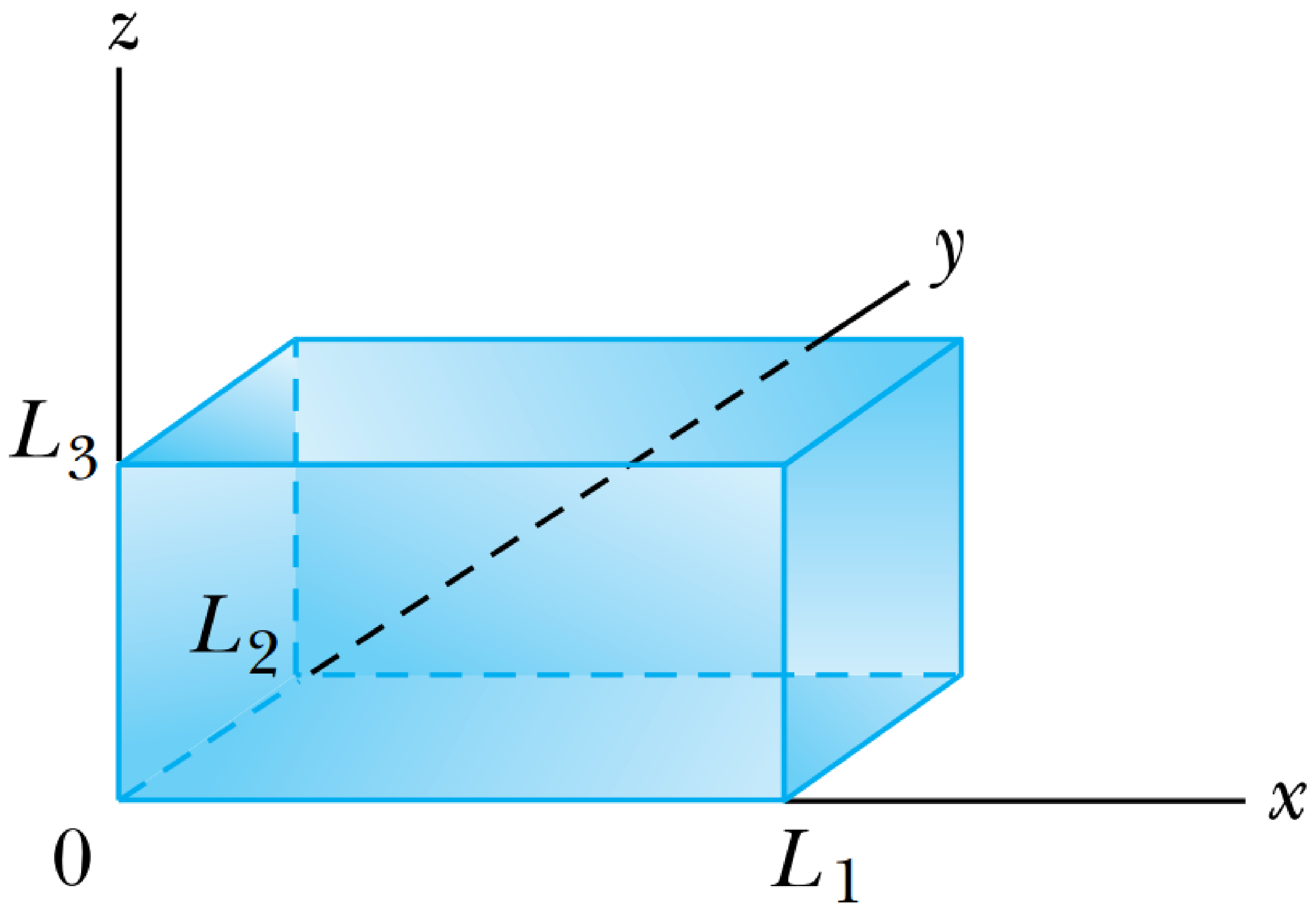
Particle in a Box



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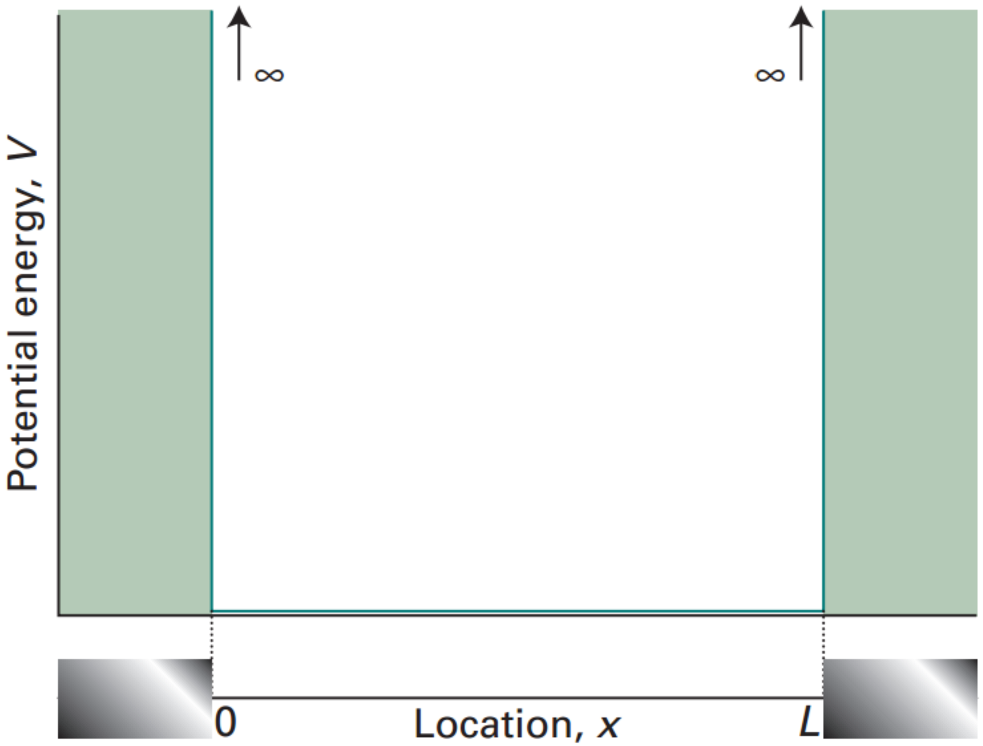
Boundary conditions:

$$\text{(i)}\ \psi(x=0)=0,$$

$$\text{(ii)}\psi(x=L)=0.$$



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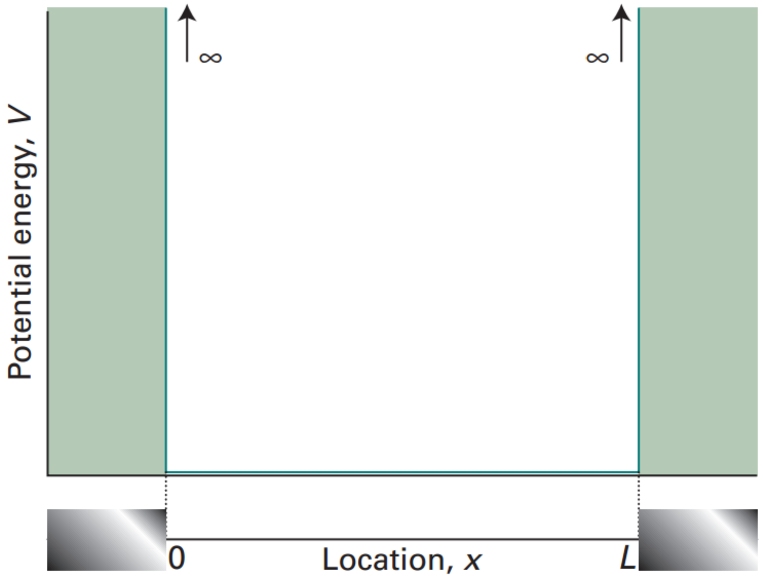
One dimensional time-independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = E\psi(x)}$$



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$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = -\frac{2mE}{\hbar^2}\psi(x)$$

$$= -k^2\psi(x)$$

$$k = \sqrt{2mE/\hbar^2}$$

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$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = -k^2\psi(x)$$

$$\boxed{\psi(x) = A\sin kx + B\cos kx}$$

where *A* and *B* are constants used to normalize the wave function.

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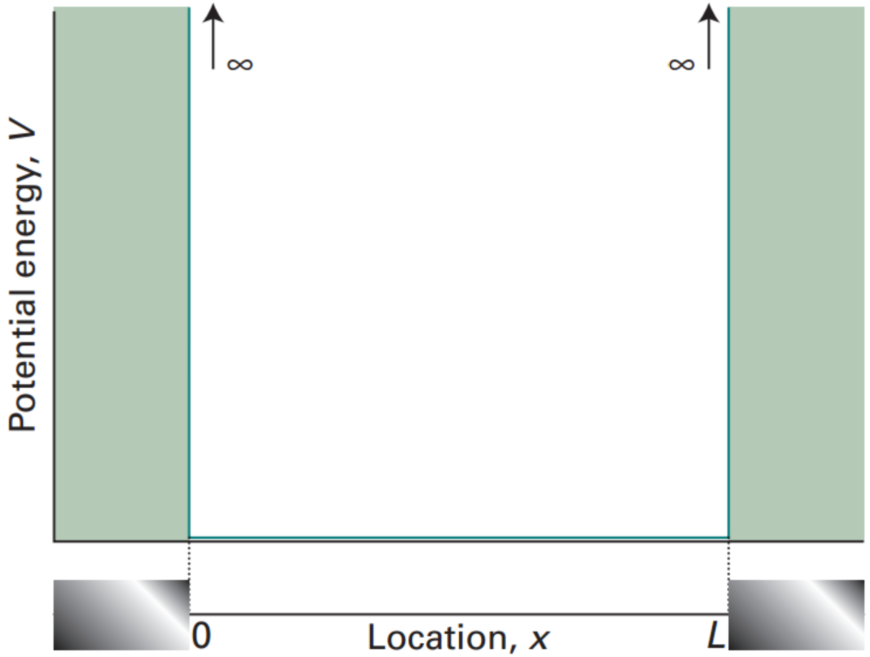

$$\boxed{\psi(x)=A\sin kx+B\cos kx}$$

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Boundary condition (i) gives

$$\psi(x=0) = A\sin 0 + B\cos 0 = 0$$

$$\boxed{B = 0}$$

Which leads us to

$$\boxed{\psi(x) = A\sin kx}$$

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From boundary condition (ii) we get

$$\psi(x = L) = A \sin kL = 0$$

As $A = 0$ leads to a trivial solution, we must have

$$\sin(kL) = 0$$

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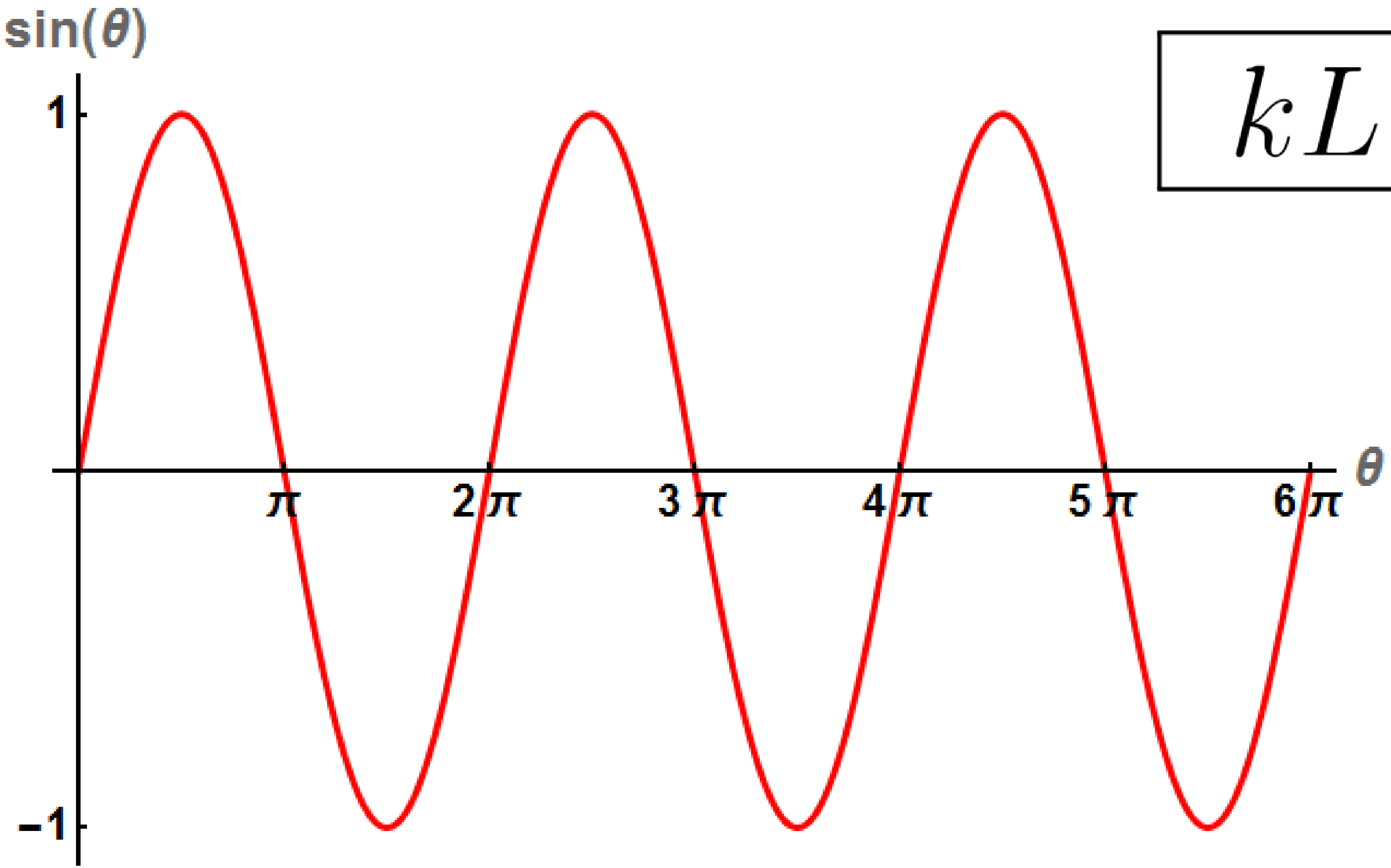

$$kL = n\pi$$

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$$\sin(kL) = 0$$

$$\boxed{kL = n\pi}$$

where n is a positive integer.

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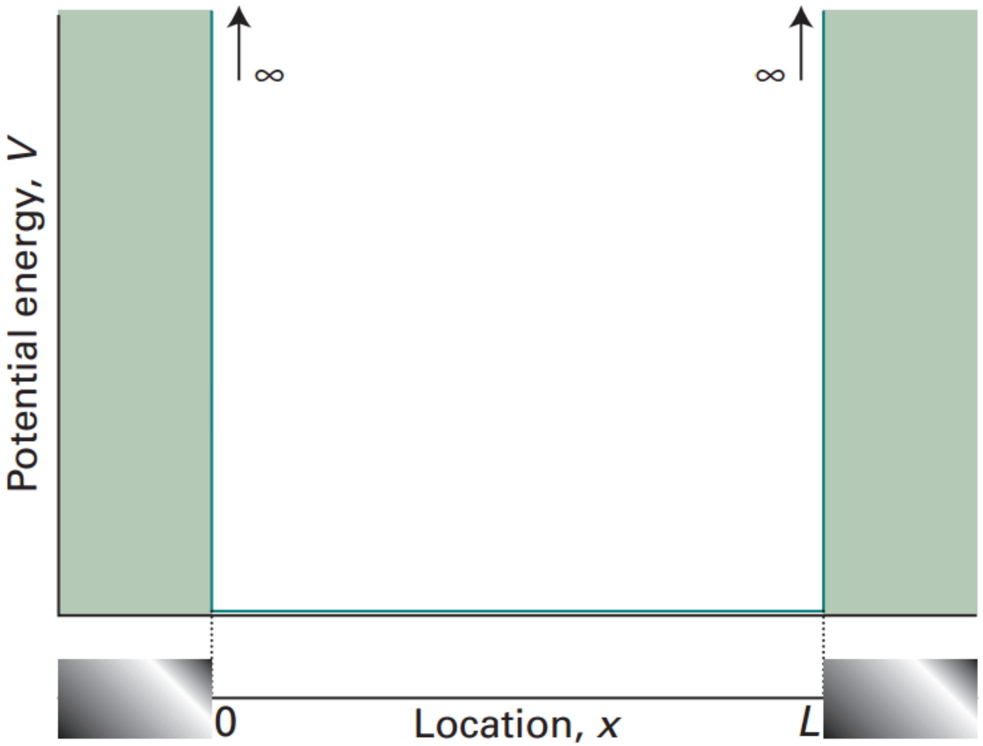
$$\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right)$$

$$(n = 1, 2, 3, 4, \ldots)$$



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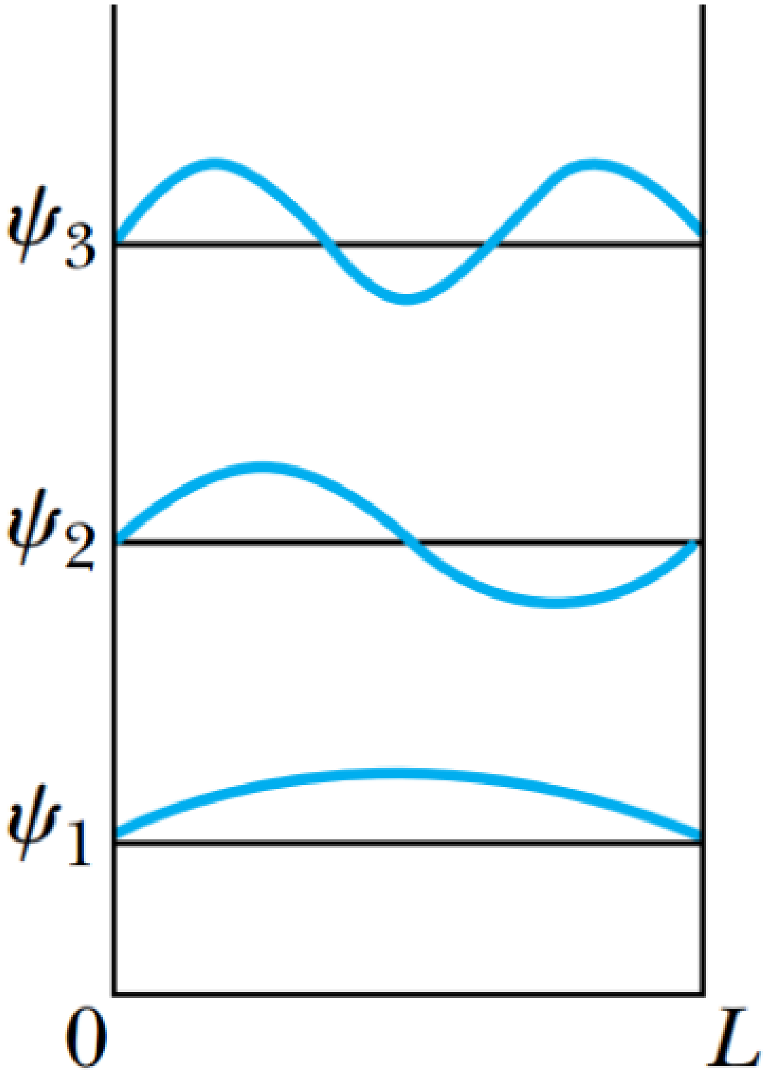

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$$k = \sqrt{2mE/\hbar^2}$$

$$kL = n\pi$$

$$\boxed{E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}}$$

$$(n = 1, 2, 3, 4, \ldots)$$

Quantized energy levels

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Infinite Square-Well Potential

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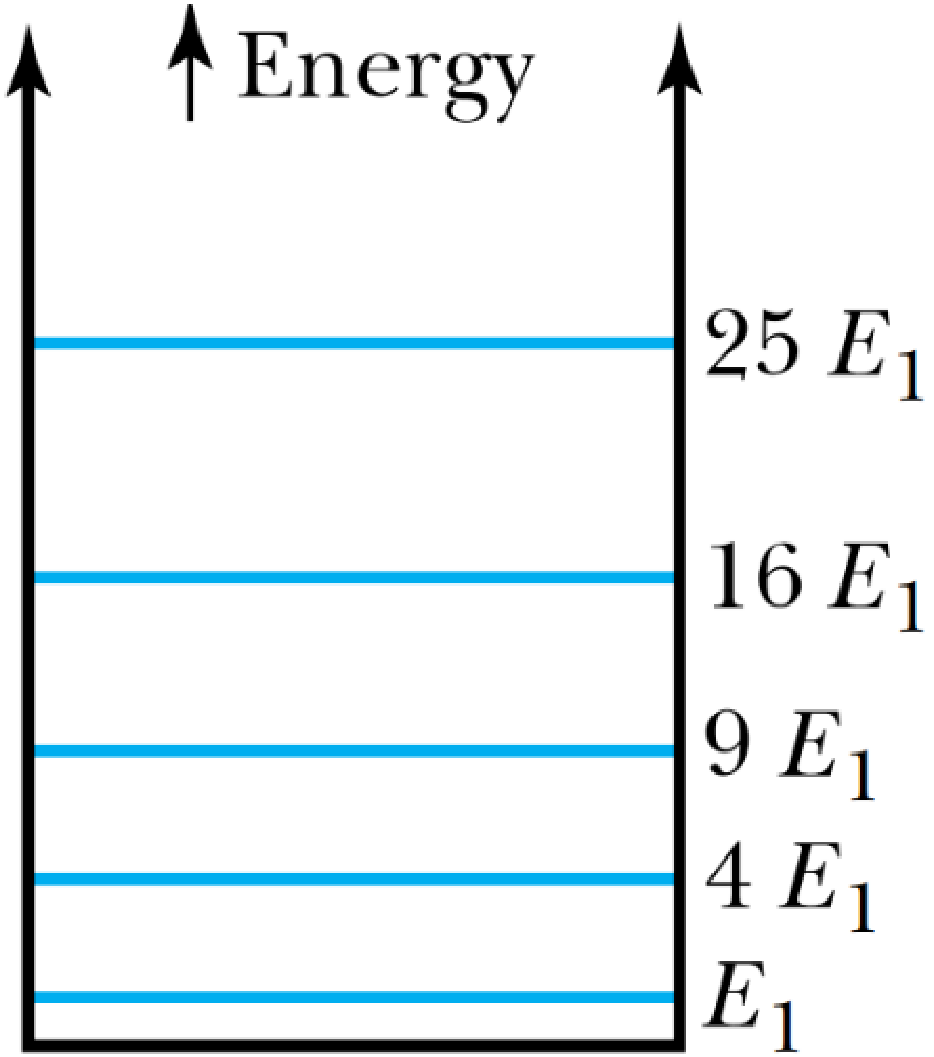
$$(n = 1, 2, 3, 4, \ldots)$$



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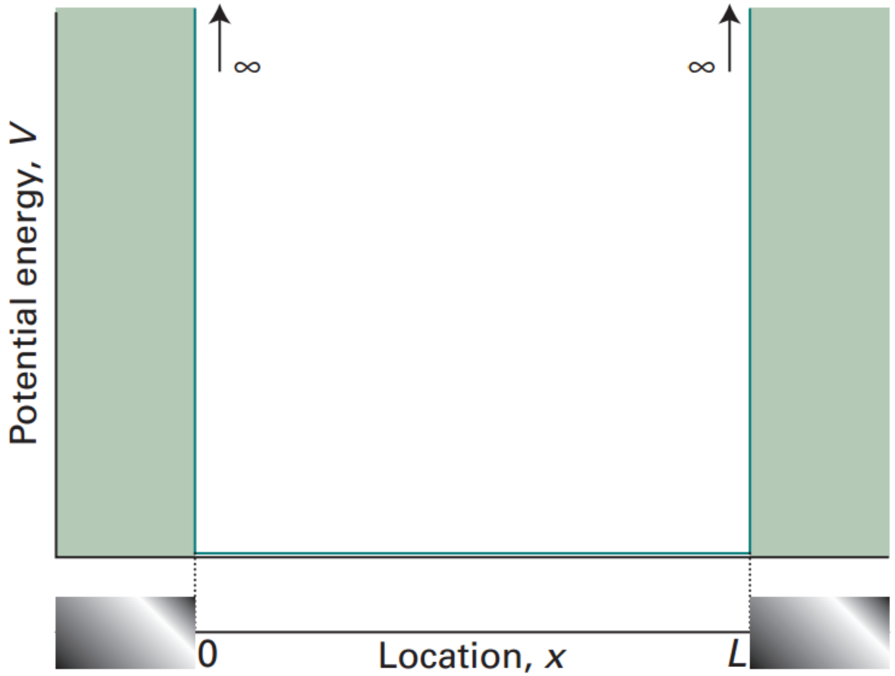

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